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THESIS

RELATIVE POSITIONING
OF
OCEAN BOTTOM BENCHMARKS

by

Feng-Yu Kuo

December 1985

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Transponder arrays such as the one deployed can be used for solving many types of problems in sea floor engineering, which will be of increasing importance in the future.

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Relative Positioning
of
Ocean Bottom Benchmarks

by

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Lieutenant Commander, Republic Of China Navy
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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN HYDROGRAPHIC SCIENCES

from the

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December 1985

ABSTRACT

A sea floor benchmark experiment was conducted in an area about 16 nmi west of Pt. Lobos, California ($36^{\circ}30'N \times 122^{\circ}17'W$) during 18-22 May 1985.

Two baseline-crossing methods were used to determine the relative positions of acoustic bottom transponders. The method of least-squares adjustment was used to analyze the data. Relative position determination of the transponder array is discussed and recommendations are made for further improvement. The advantage of these methods is their simplicity. Their disadvantage is the relatively large amount of ship time they require to achieve acceptable accuracies.

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I. INTRODUCTION

A. BACKGROUND

In today's world, the ocean scientist and engineer need increased accuracy for marine positioning to accomplish their work. The establishment of a system for the precise determination of positions on the deep seafloor is one of the most fundamental challenges of working in the open sea.

A seafloor benchmark positioning system is proposed which can determine accurately the location of objects on the sea floor (Saxena, 1974). Such precisely located benchmarks can later be used in turn to delineate offshore property lines and national boundaries. They are also useful in solving problems in seafloor engineering, in plate tectonic studies and in connection with bore-hole reentry associated with off-shore oil recovery.

The objective of this study is to demonstrate the feasibility of establishing such seafloor benchmarks.

B. DESCRIPTION

Early experiments associated with seafloor acoustic transponder arrays were carried out by Hart (1967), Haehnle (1967), Fubara and Mourad (1972), and McKeown (1975), who used a baseline crossing method to solve for the distances between transponders. "Baseline" in this thesis means the line between the projections of two bottom-mounted acoustic transponders onto the sea surface.

The baseline accuracies obtained by Hart (1967), McKeown (1975), and Fubara and Mourad (1972) were ± 15.7 m for baselines of 5509 m, ± 3.1 to ± 4.2 m over baselines of 6373 to 7219 m and ± 15.5 m over baselines of 9364 m, respectively.

The first two of the above authors did not use a least squares method for data analysis.

There are three parts to the determination of the absolute position of an ocean-bottom transponder array using satellites. They are (Figure 1.1):

1. To determine the geodetic position of the ship's receiving antenna from the Global Positioning System (GPS);
2. To convert the geodetic position of the antenna to the ship-mounted acoustic transducer; and
3. To determine the geodetic position of the acoustic transponder array on the sea floor using a ship-mounted acoustic transducer.

This thesis is concerned with the relative position of a bottom-mounted transponder array with regard to the ship.

C. DESCRIPTION OF THE SYSTEM

Acoustic transponders manufactured by Oceano Instruments were used for this project. They were deployed by the R/V Acania. Figure 1.2 shows the configuration of the Oceano system used. Their principal features are:

1. AM-121 : (Acoustic Module)

The hydrophone was fixed amidships to the port side at a depth 2.95 m.

- Transmission frequencies : 8 to 16 kHz.
- Reception frequencies : 8 to 16 kHz.

2. RT-121 : (Recoverable Transponder)

- Frequencies : 8 to 16 kHz.
- Delay time : 15.0 ± 0.1 ms.

3. TT-201 : (The Acoustic Telecommand Module) has the following functions:

- Code acoustic signal transmitted by either the acoustic module or UQC transducer.
- Monitors range to desired transponder.

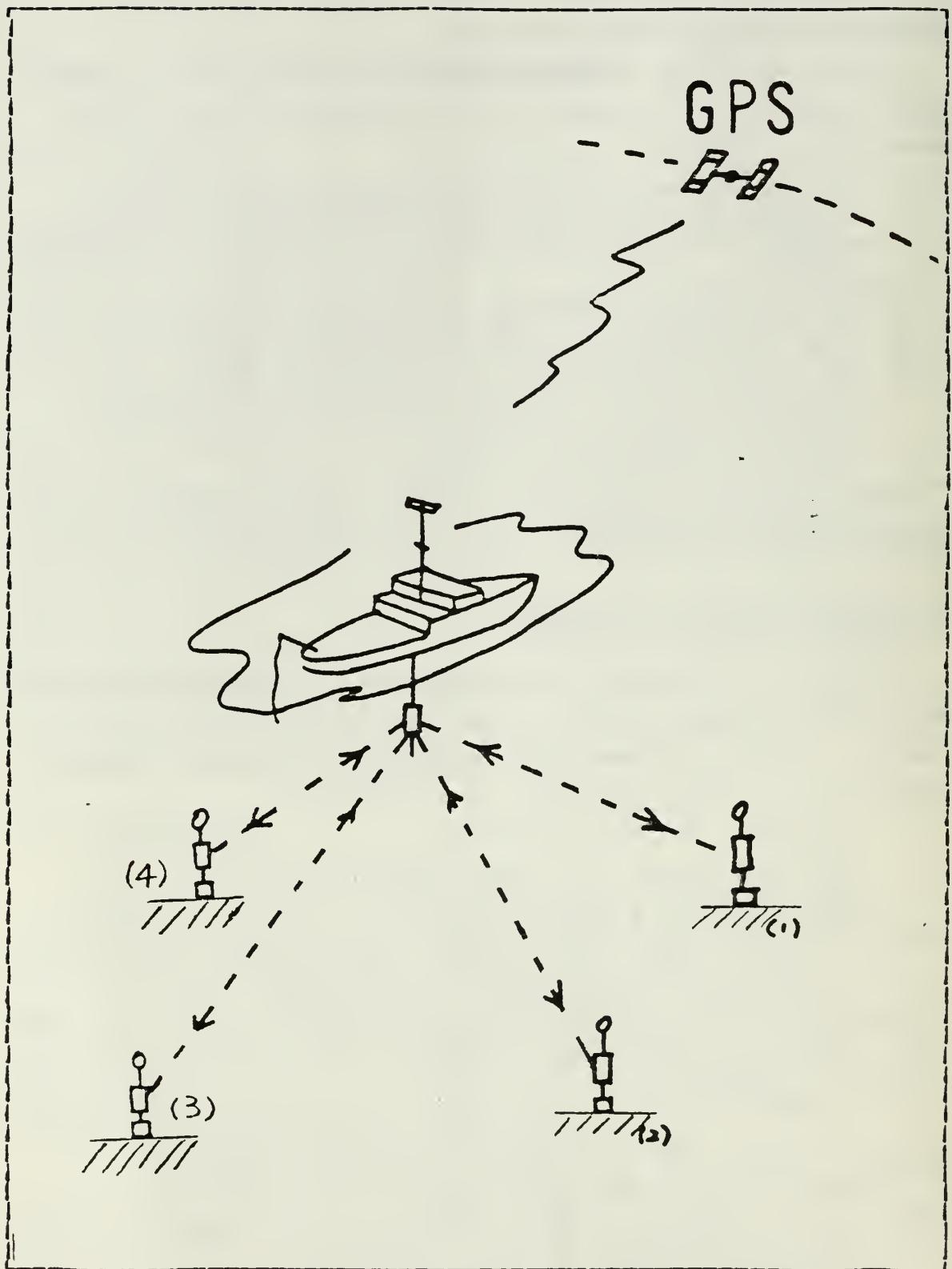


Figure 1.1 The Configuration of Bottom Transponder Array
Shown with Ship and GPS Satellite.

- Monitors reception of code by transponder.
4. IM-100 : (Data Interface Module)
- Accepts up to 16 data inputs.
 - Max. converted data format : 7 digits without sign or 6 digits plus sign.
 - Computer interrupt request possible through any input.
 - Each channel may be read separately.
5. RM-201 : (Rangemeter)
- Transponder turnaround delay offset : 0 to 99.99 ms.
 - Mean velocity of sound input : 0 to 9999.9 units per second.
 - Standard reception filters : 8 to 16 kHz (0.5 kHz step)
 - Bandwidth : 200 Hz.

Critical to the experiment are the following:

1. The transducer (a part of the AM-121) was 2.95 m below the surface, and the transmission and reception frequencies were 8 to 16 kHz.
2. The system delay on the RM-201 was set at 15 milliseconds, i.e., the time lapse between reception and transmission of a signal.
3. Velocity on the RM-201 was set to 1480 m/s.
4. Interrogation period was set at 10 s.
5. Interrogation frequency (kHz) was set 15 kHz.

The numerical constants of (3) and (4) were used for the survey. Details of the transponders are given by Oceano Instruments (1984).

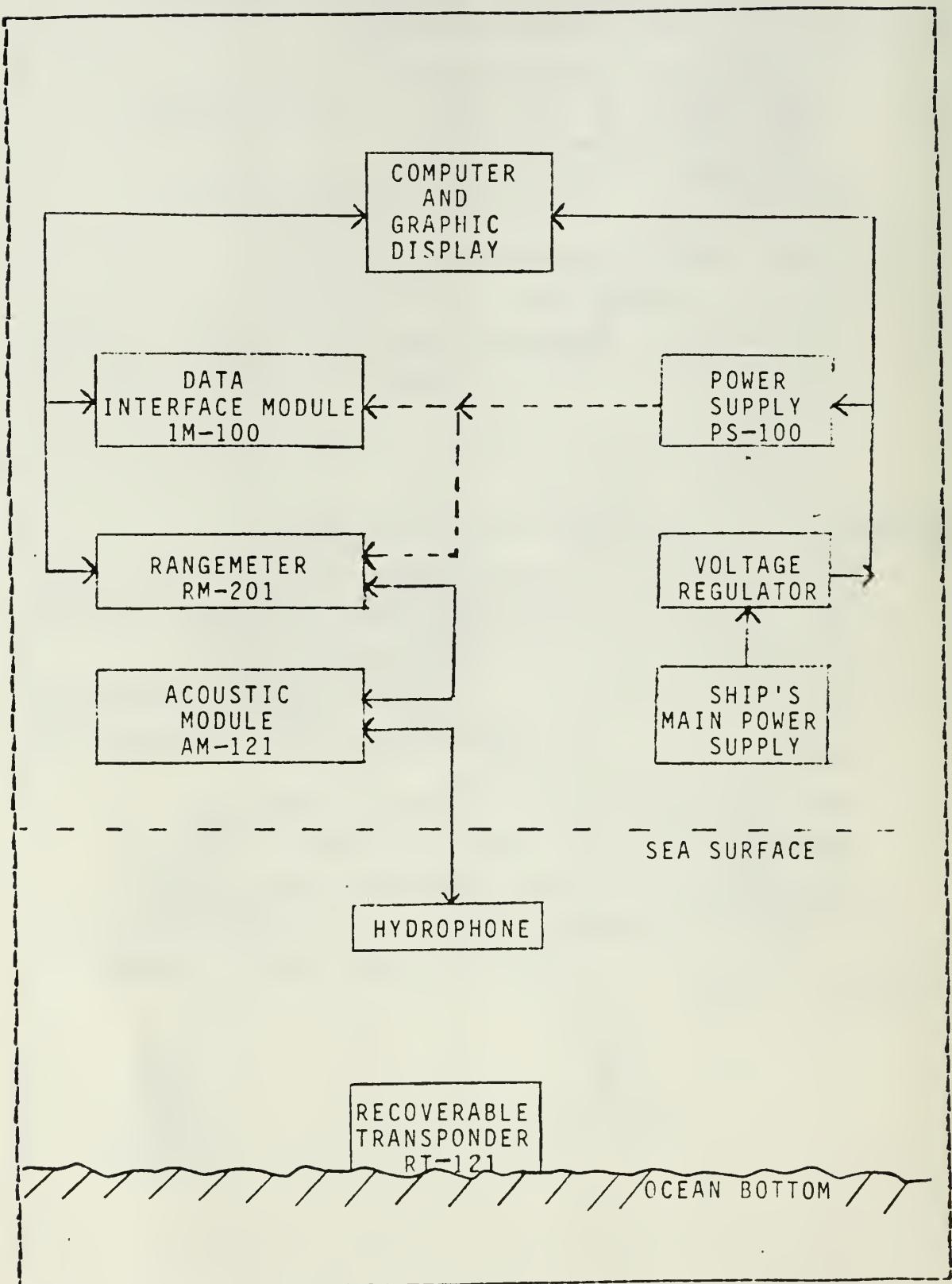


Figure 1.2 Configuration of The Oceano Systems Used.

III. THE OPERATIONAL PROCEDURE FOR IMPLANTATION OF A TRANSPONDER ARRAY

A. SITE SELECTION

The main criterion for the site of the experiment was that it be relatively flat over an area approximately 2 km x 2 km. Since there were no existing charts detailed enough, a reconnaissance survey was performed by the R/V ACANIA on 10 and 11 April 1985. Some 10 north-south sounding lines and 1-cross line were run with 1-nmi spacings and 5-min position fixes.

The area chosen for the experiment (Figure 2.1) is approximately 24 nmi west of Pt. Lobos, California, in water approximately 1600 to 2000 m deep. It is the area from $36^{\circ}29.5'$ to $36^{\circ}31'N$ and $122^{\circ}16'$ to $122^{\circ}17.8'W$. Mini-Ranger stations (Figure 2.2) were set up at Point Sur ($36^{\circ}18'20.279''N$ X $121^{\circ}53'56.179''W$) and Carmel ($36^{\circ}33'49.176''N$ X $121^{\circ}53'48.358''W$) for additional ship position control.

B. IMPLANTATION PROCEDURE

The transponders were deployed at pre-selected sites using a buoy-first, anchor-last, technique. The anchors used were railroad wheels which weigh about 290 kg in water. The mooring cables between the anchors and the transponders and between the transponders and the buoys are each 8-m long. The transponders weigh about 46 lbs in water, and each buoy has 55 lbs positive floatation. Three 17-in Benthos glass sphere floatation buoys were attached to each transponder by means of 5/32-in stainless cables 8-m long. Although 2 spheres would have provided sufficient buoyancy

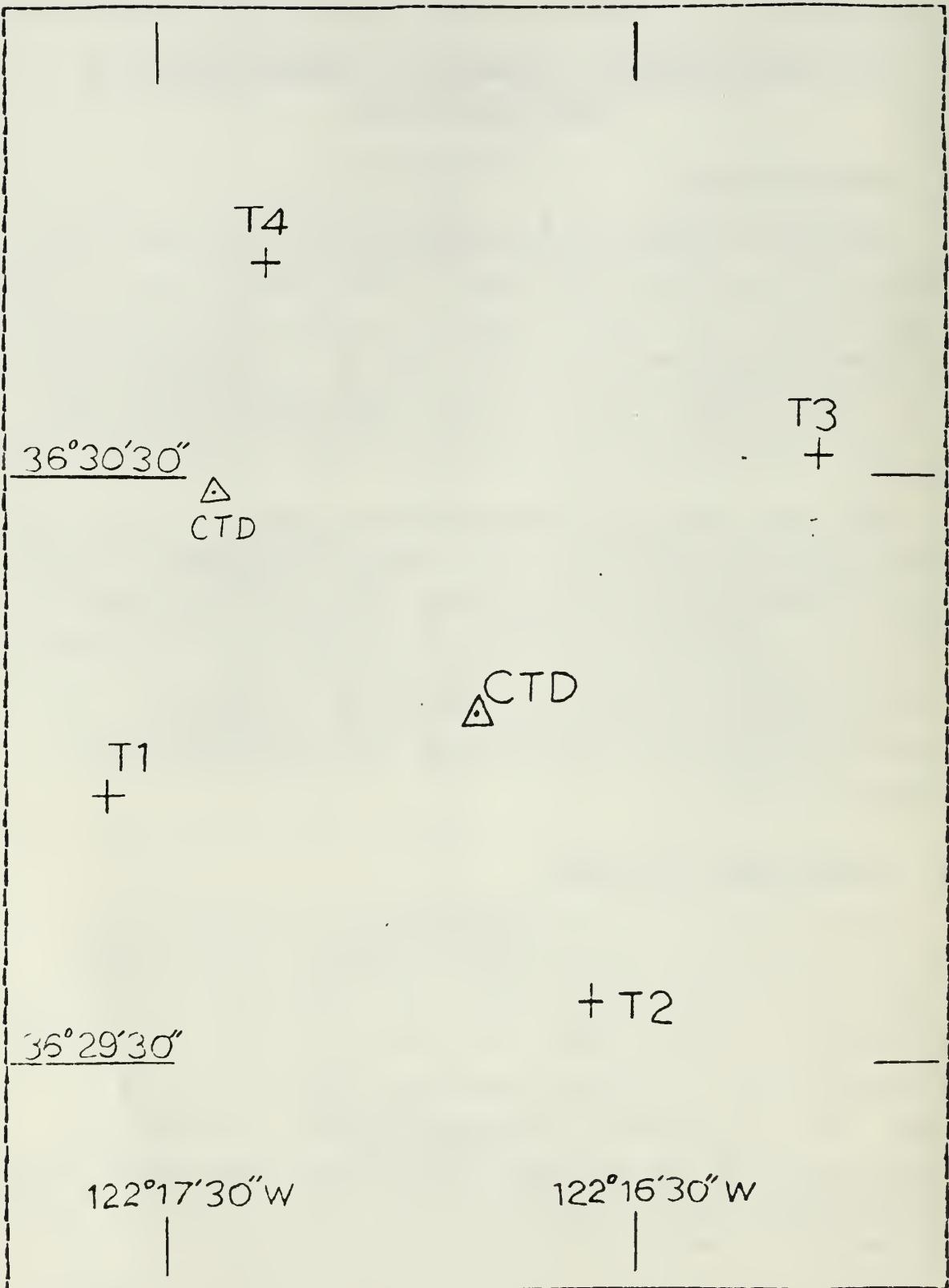


Figure 2.1 The Project Area.

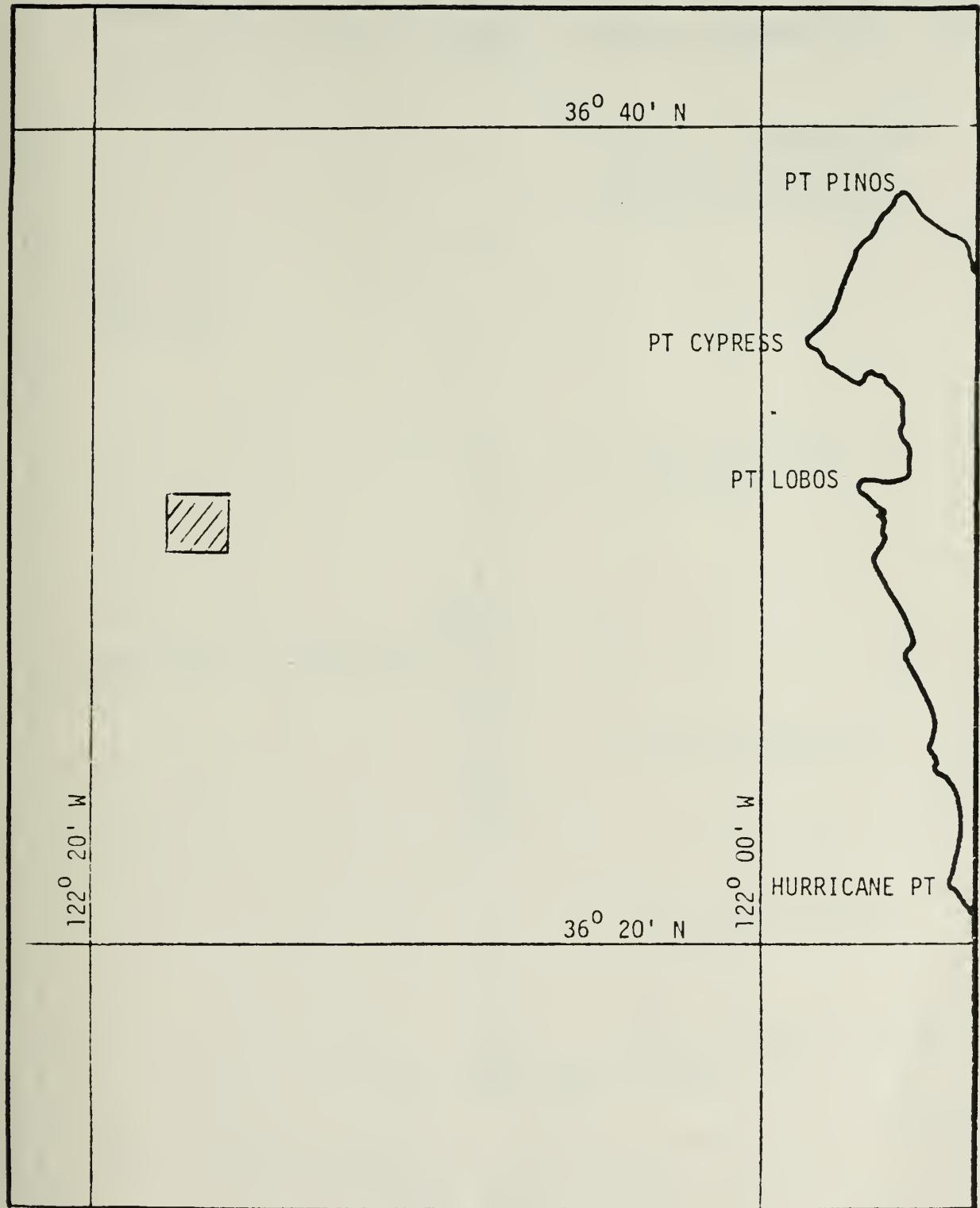


Figure 2.2 The Project Area with Shoreline.

for recovery, a third was used to provide additional tension (about 50 kg total) to reduce the motion of the transponders. The mooring system is shown in Figure 2.3.

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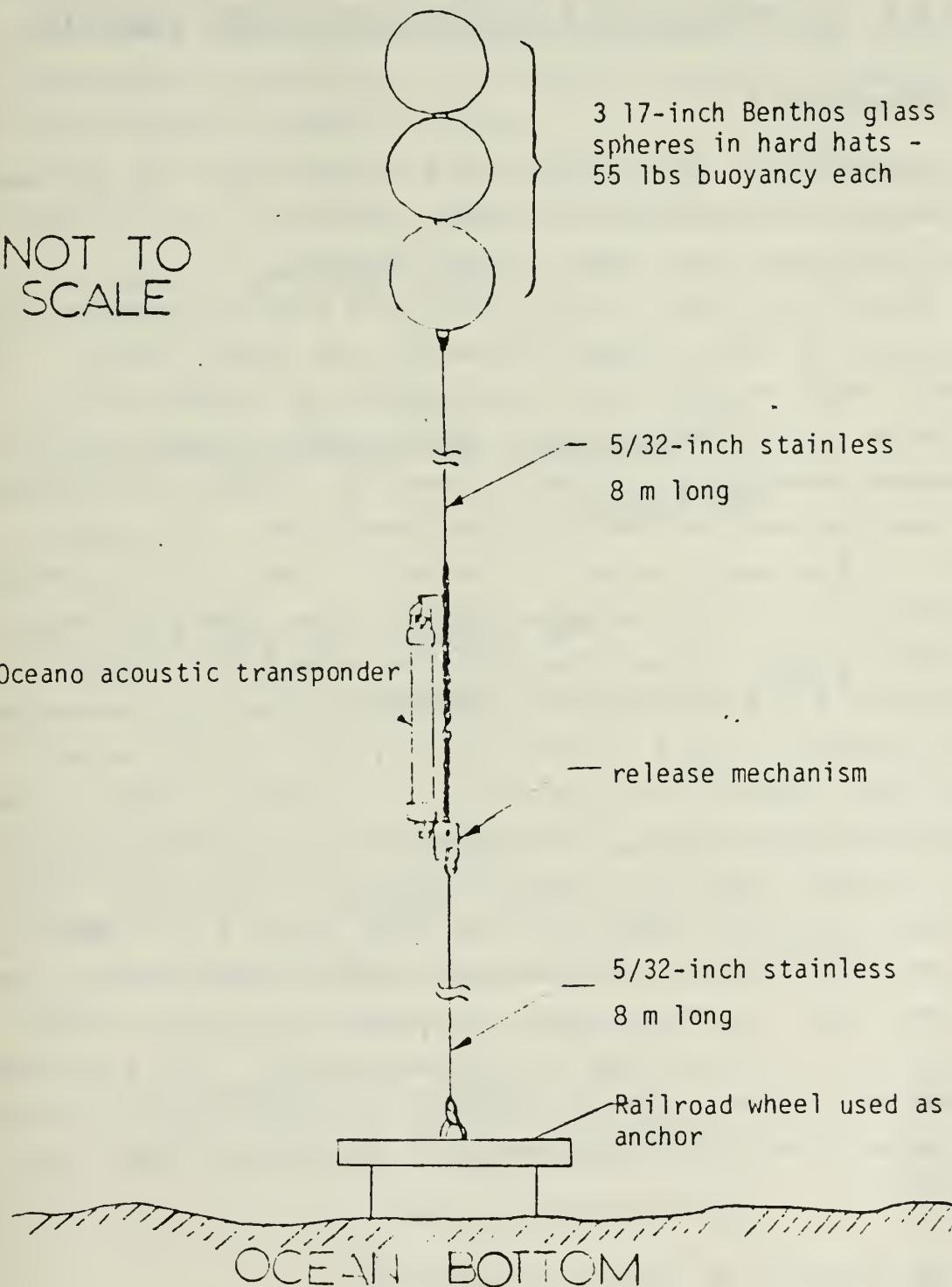


Figure 2.3 Schematic of Bottom Transponder Mooring.

III. DETERMINATION OF HARMONIC MEAN SOUND VELOCITY

A. INTRODUCTION

Historically, echo soundings have been made by assuming a constant, approximately average, value for the velocity of sound throughout the water column, usually 4800 ft/s (=1463 m/s) or 1500 m/s, and then correcting for variations of this assumed value for the actual water column. Echo sounders are time-measuring devices. The sounder's acoustic transducer emits a sound pulse which is reflected upwards by the bottom-mounted acoustic transponder and received back at the surface transducer. The time of travel of the sound pulse is divided by two, and this value is multiplied by the assumed harmonic mean speed of sound in sea water, thus, giving the distance according to the expression $D = V t$. The echo sounder makes this transformation electronically or mechanically within the device itself and displays the distance in the water between transducer and transponders. This distance is not equal to the true distance, since the assumed harmonic mean sound speed generally does not equal the true mean speed for a particular depth. To determine the true harmonic mean sound speed one must know the sound speed throughout the water column and apply it to solve for the harmonic mean. For a distance measurement to be accurate, precise measurements of time and the harmonic mean speed of sound in the water column are mandatory.

B. THE VELOCITY OF SOUND IN THE OCEAN

The velocity of sound in sea water is a function of temperature, pressure, and salinity: $V = V(T, S, P)$. Because

the sound velocity is not constant with position, sound rays are refracted according to Snell's law.

We have followed the standard practice of the U.S. National Ocean Service (NOS) to use Wilson's (1960) equation for the speed of sound as a function of salinity, temperature and pressure (Umbach, 1976):

$$V = 1449.14 + V_T + V_P + V_S + V_{STP} \quad (3.1)$$

where

$$\begin{aligned} V_T = & 4.5721T - 4.4532 \times 10^{-2} T^2 - 2.6045 \times 10^{-4} T^3 \\ & + 7.9851 \times 10^{-6} T^4 \end{aligned} \quad (3.2)$$

$$\begin{aligned} V_P = & 1.60272 \times 10^{-1} P + 1.0268 \times 10^{-5} P^2 \\ & + 3.5216 \times 10^{-9} P^3 - 3.3603 \times 10^{-12} P^4 \end{aligned} \quad (3.3)$$

$$V_S = 1.39799(S-35) + 1.69202 \times 10^{-3} (S-35)^2 \quad (3.4)$$

$$\begin{aligned} V_{STP} = & (S-35)(-1.1244 \times 10^{-2} T + 7.7711 \times 10^{-7} T^2 \\ & + 7.7016 \times 10^{-5} P - 1.2943 \times 10^{-7} P^2 + 3.1580 \times 10^{-8} PT \\ & + 1.5790 \times 10^{-9} PT^2) + P(-1.8607 \times 10^{-4} T \\ & + 7.4812 \times 10^{-6} T^2 + 4.5283 \times 10^{-8} T^3) \\ & + P^2(-2.5294 \times 10^{-7} T + 1.8563 \times 10^{-9} T^2) \\ & + P^3(-1.9646 \times 10^{-10} T). \end{aligned} \quad (3.5)$$

In these equations the absolute pressure, P , is expressed in kg/cm^2 , temperature, T , is in $^\circ\text{C}$, salinity, S , is in g/kg , and sound velocity, V , is in m/s . For these relations for V the standard deviation from the mean is 0.30 m/s for all data obtained in the ranges $-4 \leq T \leq 30$ $^\circ\text{C}$, $1 \leq P \leq 1000$ kg/cm^2 , and $0 \leq S \leq 37$ g/kg (Wilson, 1960, p.1357).

C. HARMONIC MEAN SOUND VELOCITY

The slant range from a ship to a transponder is determined by measuring the transit time, t , of an acoustic pulse and converting it to distance using an appropriate value for sound speed. If the mean sound velocity, V , in the water column is known, the distance, L , from transducer to transponder can be computed by:

$$L = (t/2) \bar{V} \quad (3.6)$$

For our small project area the distances between transponders are less than 2 nmi, and it is assumed that there is no horizontal sound speed variation, and only corrections for vertical sound speed variation are considered. For a one-way travel time, T , through a water column of depth, Z , the mean sound velocity is:

$$\bar{V} = Z / T \quad (3.7)$$

The meaning of "mean sound velocity" is neither "mean velocity from the surface to the stated depth" nor the "mean value for the velocity of sound through the vertical water column". According to Equation (3.7) the mean velocity is obtained from the true depth divided by the one way travel time.

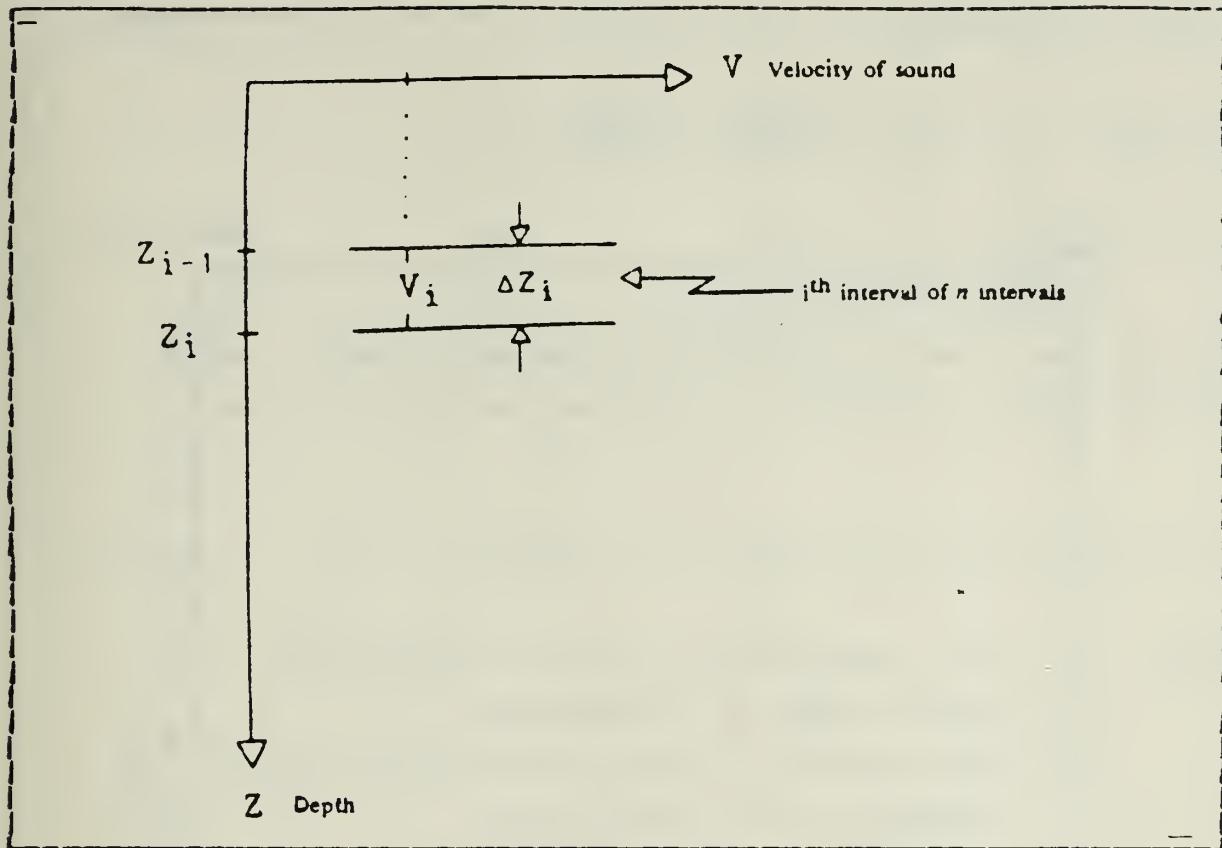


Figure 3.1 A Layered Ocean Where $v_i = v_i(z)$ Is the Sound Speed in the i^{th} Layer and z Is Depth.

Consider now a layered ocean having a different sound speed in each layer. Figure 3.1 shows a series of finite layers of thickness Δz_i , each with an associated sound speed, v_i . The time interval, ΔT_i , for the sound wave to pass vertically through the i^{th} layer is :

$$\Delta T_i = \Delta z_i / v_i \quad (3.8)$$

Summing the time intervals for all n layers in a vertical water column from surface transducer to bottom transponder ,

$$T = \sum_{i=1}^n \Delta T_i = \sum_{i=1}^n (\Delta z_i / v_i) \quad (3.9)$$

For a continuous function, $v = v(z)$, this becomes

$$T = \int_0^Z dz/V(z) \quad (3.10)$$

Equation (3.7) may now be written :

$$\bar{v} = \frac{Z}{\int_0^Z \frac{dz}{V(z)}} \quad (3.11)$$

This is the integral form of the harmonic mean. Using Simpson's Rule, Equation (3.10) can be rewritten as :

$$T = (1/v_0 + 4/v_1 + 2/v_2 + 4/v_3 + \dots + 1/v_z) d/3 \quad (3.12)$$

where $d = D/n$ (magnitude of depth increments)

D = bottom depth at transponder

n = even number of depth increments between transducer and transponder

v_j = measured sound speed at depth j .

Since the harmonic mean sound velocity is obtained by Equation (3.11), the slant range from transducer to acoustic transponder can be obtained using Equations (3.10) and (3.11) :

$$L = T \bar{v}$$

CTD casts to 1400 m were made on 18 and 20 May 1985 in the project area just prior to and during our main experiment. Sound velocities and harmonic means based on the sound velocity profiles derived from the CTD casts are plotted in Figures 3.2 and 3.3 . Although the profiles shown in these figures are for different days and different places within the project area, they show very little variation in harmonic mean sound velocity, and thus support our previous assumption that the sea water is horizontally homogeneous to an acceptable degree within the study area.

18 MAY 1985

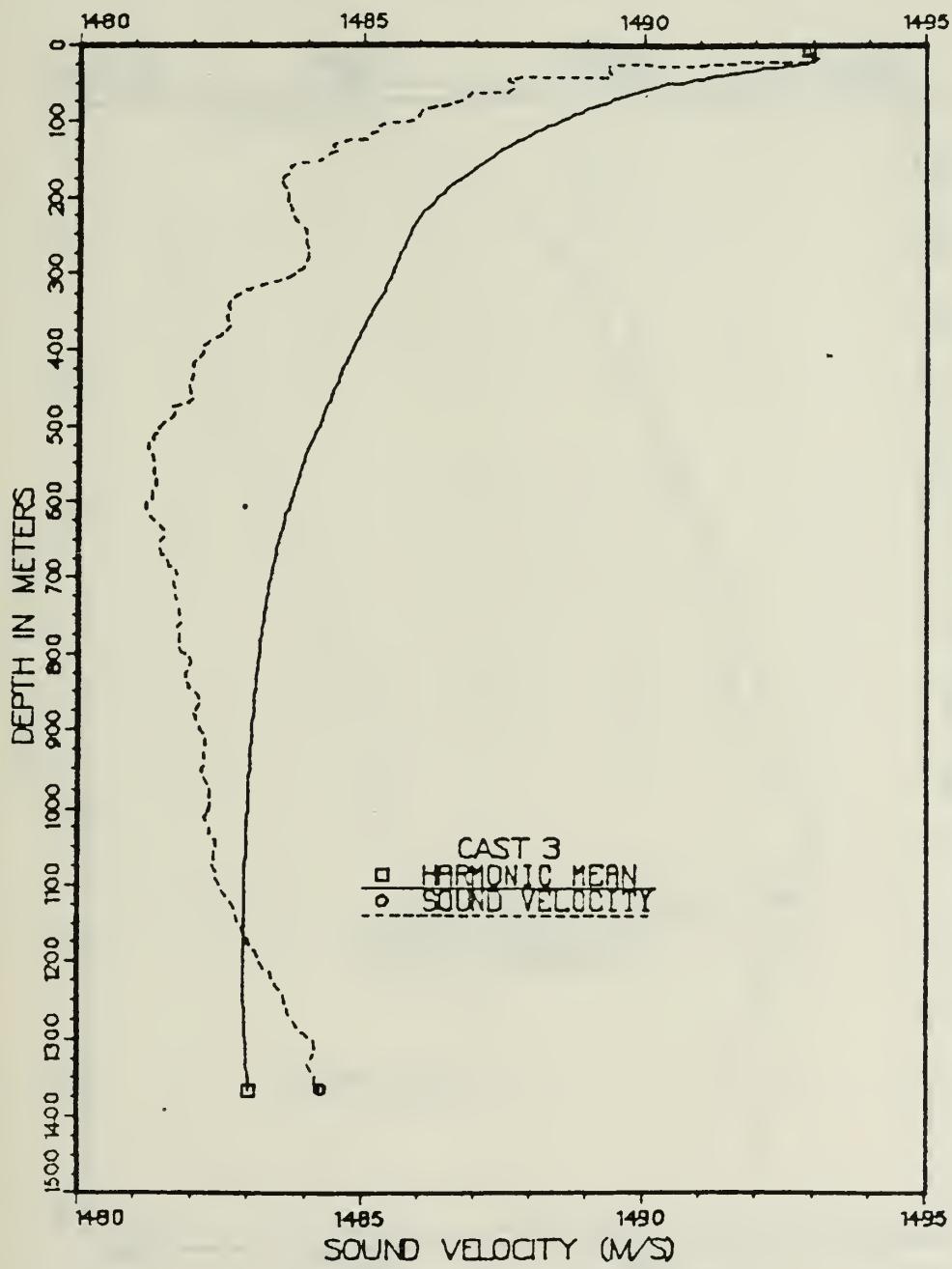


Figure 3.2 Harmonic Mean and Sound Velocity Profile 1.

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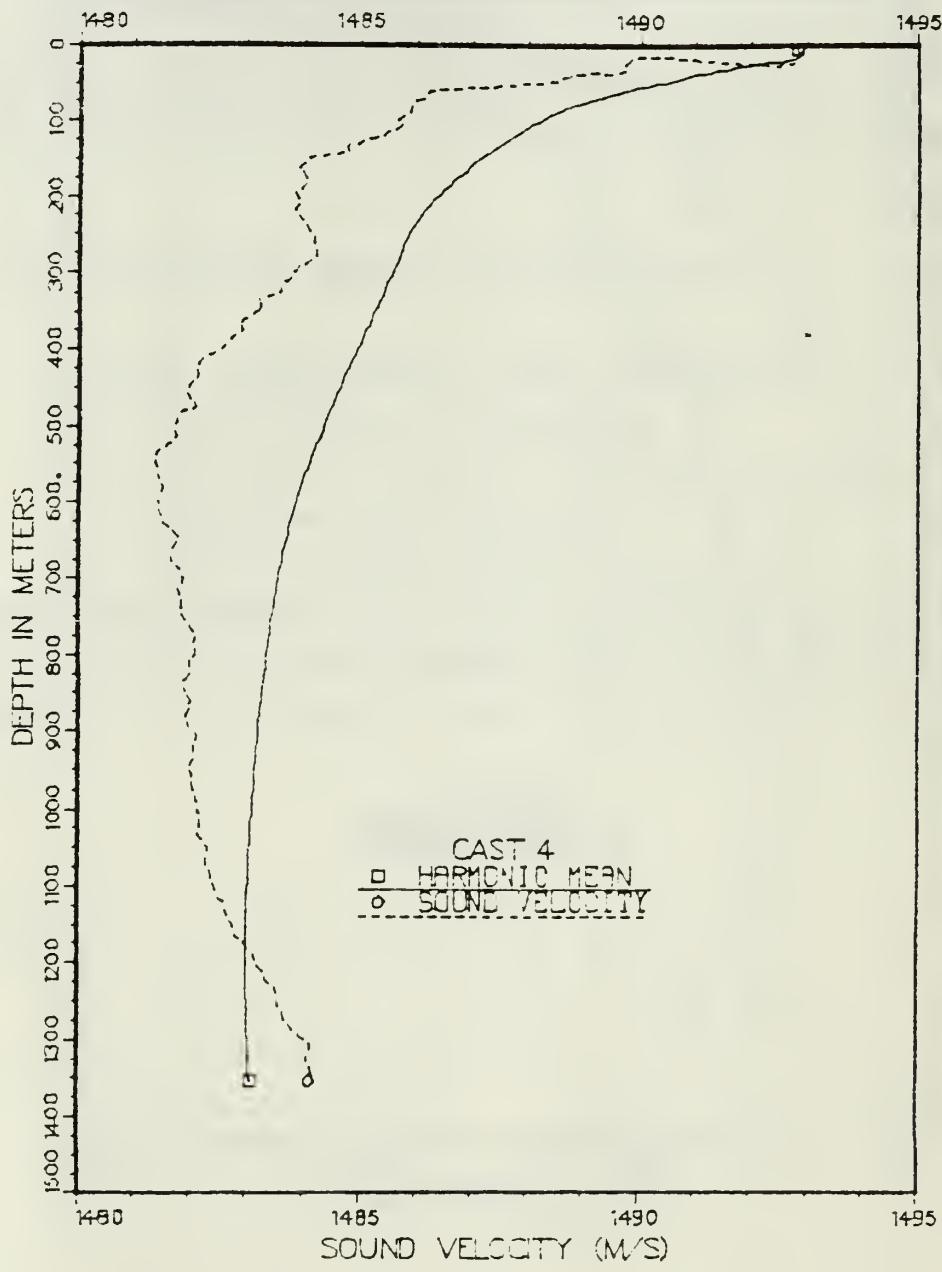


Figure 3.3 Harmonic Mean and Sound Velocity Profile 2.

HARMONIC MEAN COMPARISON

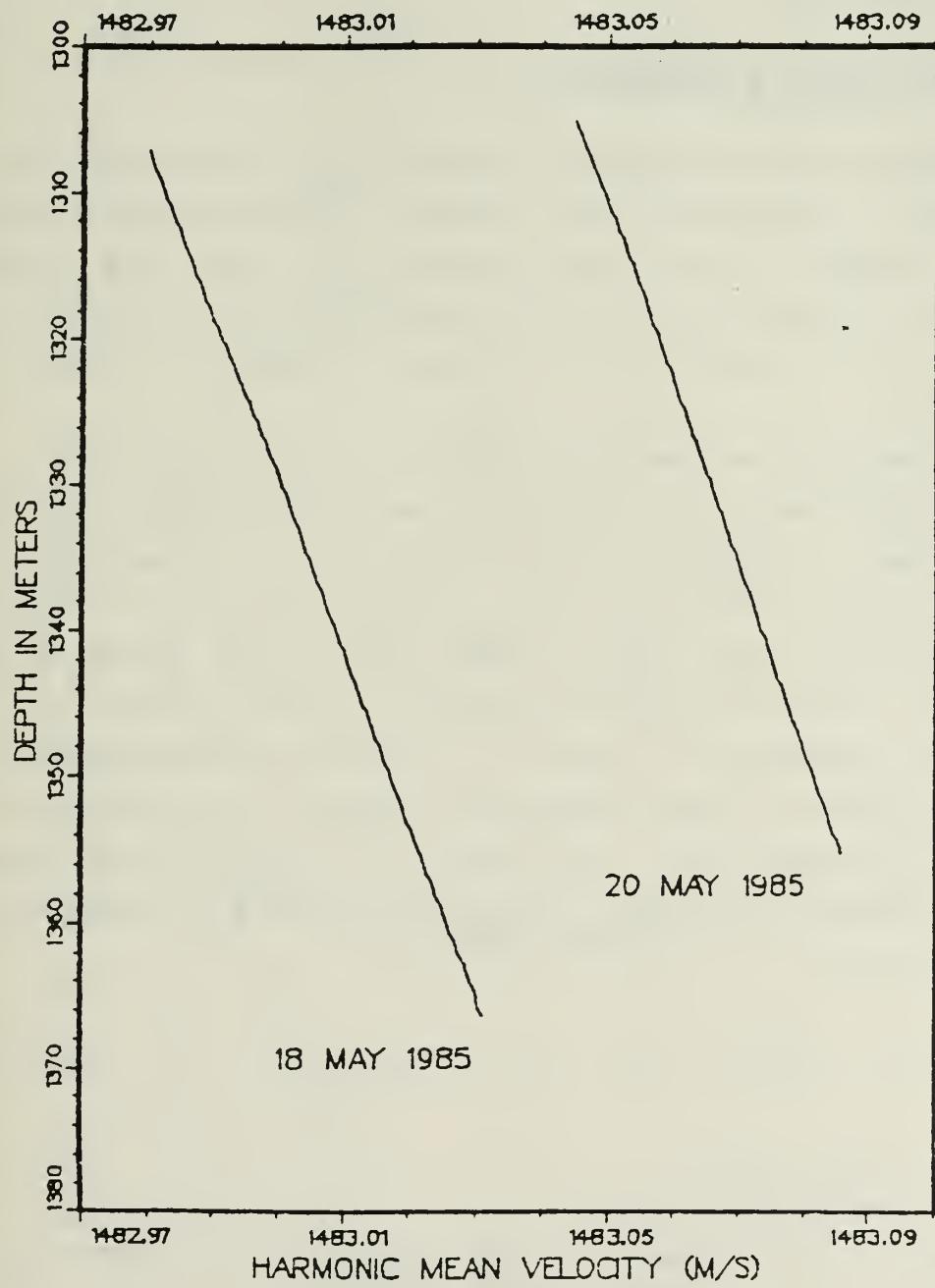


Figure 3.4 Harmonic Mean Comparison.

Actually, the difference of these two harmonic mean velocities is approximately 0.07 m/s near the bottom as is shown in Figure 3.4. The mean of these two harmonic mean sound velocity profiles is used in the range calculations of the following chapter.

D. ACOUSTIC WAVE REFRACTION

To simplify our acoustic analysis it is desirable to assume that in the study area sound follows straight paths from the surface to the ocean bottom. To test this approximation the differences in path lengths from the surface to the bottom were calculated for varies depression angles at the surface.

Profiles such as those of Figures 3.2 and 3.3 are usually simplified for analysis by separation into an appropriate number of segments each having an approximately constant gradient (Figure 3.5). If the velocity of sound changes linearly with depth, sound rays can be shown to have a constant radius of curvature (Urick, 1983, p. 124). Horizontal distance, d , depth, z , radius of curvature, R , segments of arc, s , and depression angle, A , are shown schematically in Figure 3.6. The path of a ray when the speed of sound varies with depth can be calculated by application of Snell's law:

$$\cos A_1 / C_1 = \cos A_2 / C_2 = \dots = \cos A_L / C_L \quad (3.13)$$

where C_i = sound speed

A_i = depression angle

For an initial depression angle of 30° the profile of Figure 3.5 gives the results shown in Table I.

The radius of curvature can be obtained in the following manner (Kinsler, 1982, pp399-402). Let

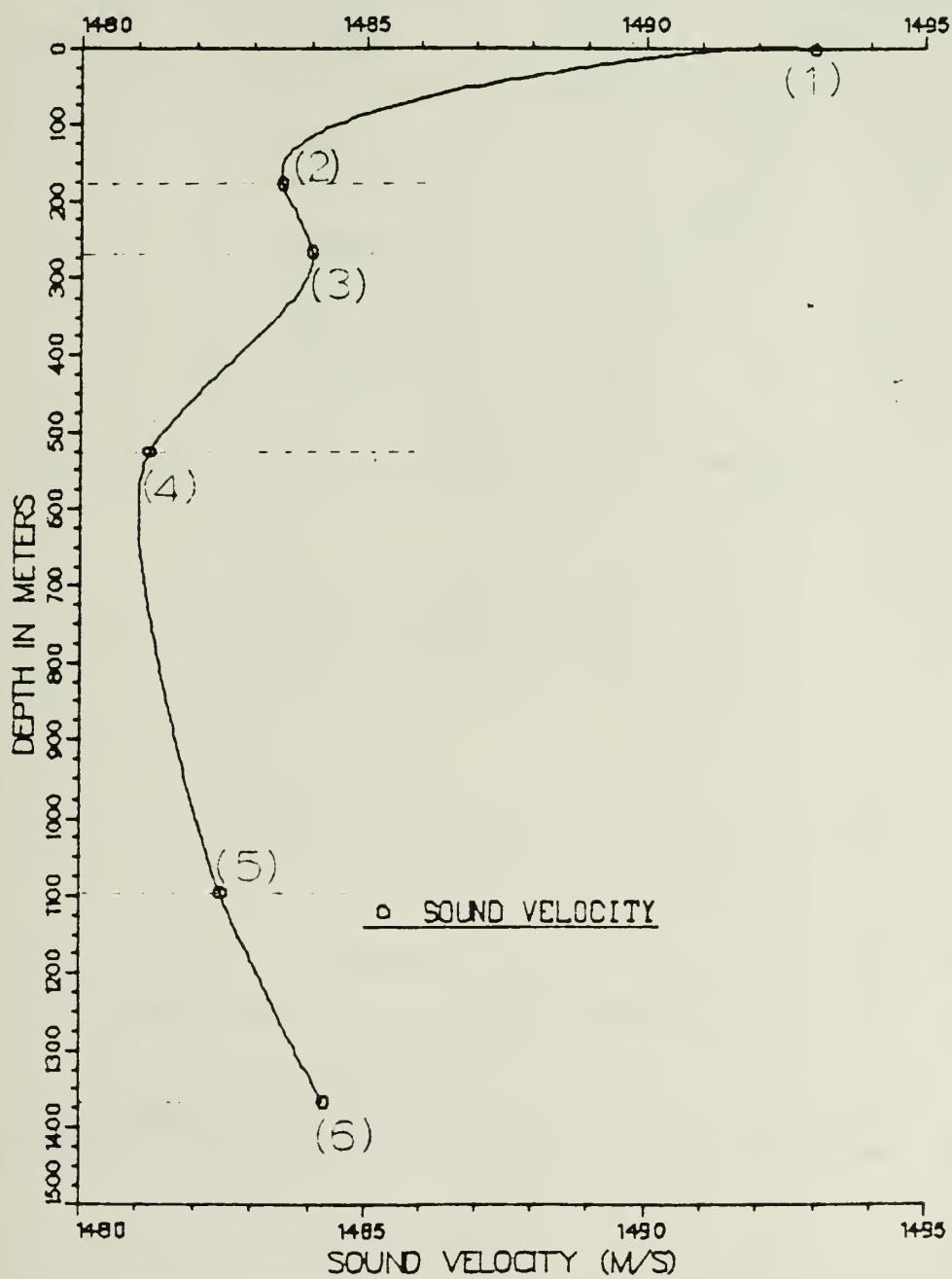


Figure 3.5 Simplified Sound Velocity Profile.

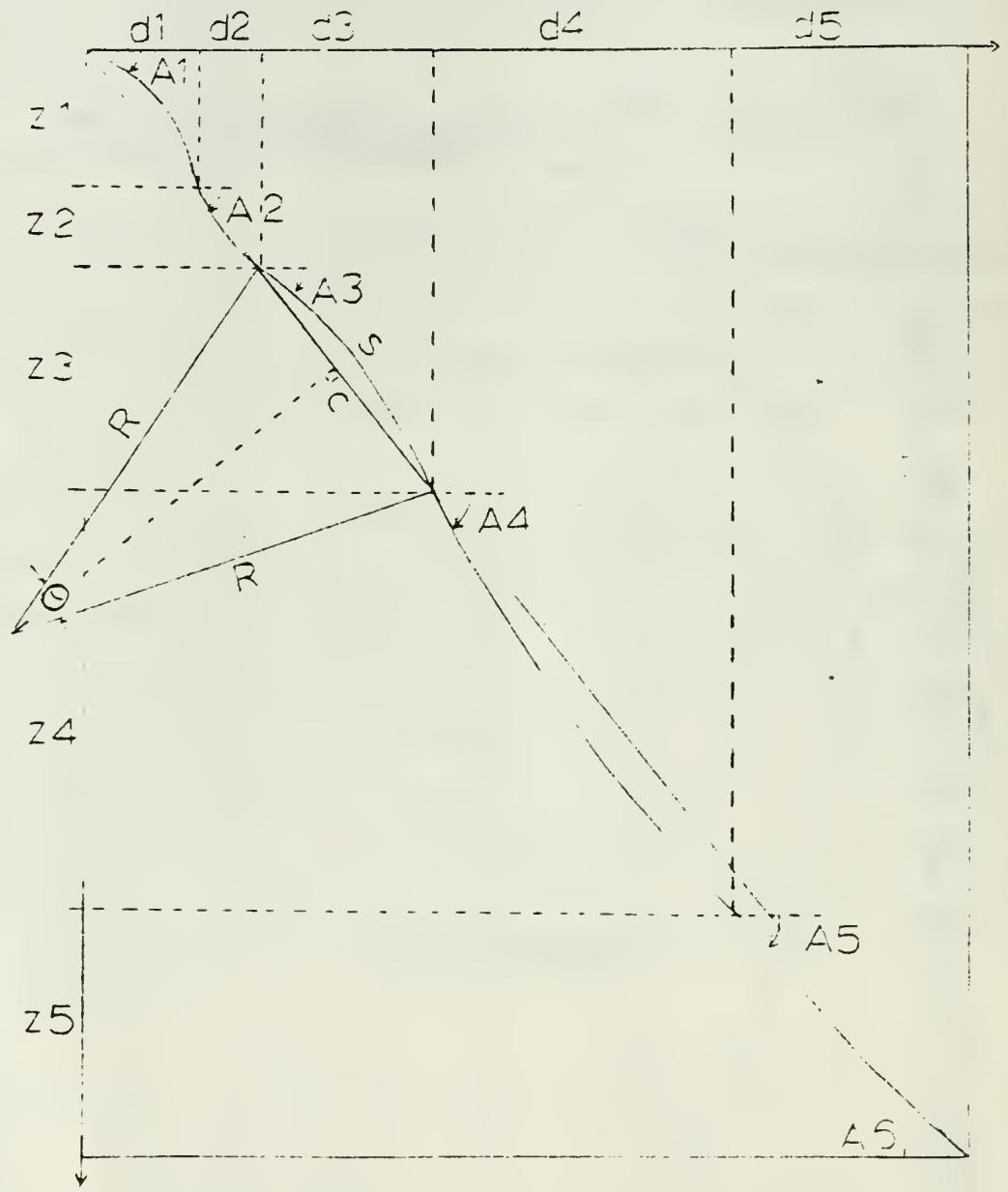


Figure 3.6 Schematic Diagram of Sound Ray Trace.

TABLE I
Depression Angle at Different Layers

Points	Sound Velocity	Depth (m)	Depression angle (deg.)
1.	1493.0	0.	30.0
2.	1483.5	176.4	30.63
3.	1484.04	266.6	30.59
4.	1481.2	526.4	30.78
5.	1482.46	1097.2	30.69
6.	1484.34	1400.0	30.57

$$g_i = (c_{i+1} - c_i) / (z_{i+1} - z_i) \quad (3.14)$$

and

$$B_i = - (c_i / g_i) \cos A_i \quad (3.15)$$

where

R_i = Radius of curvature in the i^{th} depth increment

c_i = Sound speed in the i^{th} depth region

A_i = Depression angle

g_i = Speed gradient

The change in range Δd_i and depth Δz_i are (Kinsler, 1982) :

$$\Delta d_i = -R_i (\sin A_i - \sin A_{i+1}) \quad (3.16)$$

$$\Delta z_i = -R_i (\cos A_{i+1} - \cos A_i) \quad (3.17)$$

so the chord can be obtained by two components of change in range and depth; that is,

$$D_i = (\Delta d_i^2 + \Delta z_i^2)^{1/2} \quad (3.18)$$

The angle, θ_i , is

$$\theta_i = 2\sin^{-1} (D_i / 2R_i) \quad (3.19)$$

so the arc, s_i , is:

$$s_i = R_i \theta_i \quad (3.20)$$

where θ_i is the angle in radians.

The range and depth increments give the straight line

$$L = [(\sum \Delta d_i)^2 + (\sum \Delta z_i)^2]^{1/2} \quad (3.21)$$

The ray paths can be obtained by summing arcs:

$$S = \sum s_i \quad (3.22)$$

The difference due to refraction between a sound ray's actual path and a straight line is thus:

$$E = S - L \quad (3.23)$$

where E = the error between two lines,

S = the distance along the ray path, and

L = the straight-line distance between the ship-mounted transducer and bottom transponder.

Ray-trace computations were made for various initial depression angles (at the surface) down to a depth of 1400 m (Tables II and III). Since $E \leq 1$ cm for a 30° surface

TABLE II
The Result of Ray Trace Computation

For Initial Depression Angle = 30 Degrees

Depth Increment	Gradient g	Radius R [m]	Horiz. Dist. (d) [m]	Depth z [m]	Arc s [m]
1.	0.0538	32011.36	301.72	176.4	349.5023
2.	0.0060	287966.49	152.47	90.2	177.1543
3.	0.0109	157706.64	437.85	259.8	509.1278
4.	0.0022	780984.82	960.03	570.8	1116.8993
5.	0.0062	277668.87	511.35	302.8	594.2820
Total :					
		2363.42	1400.0	2746.9658	
The Distance of Straight Line :					
The Error : 0.0096 meters					

depression angle and a 1400-m water depth, corresponding to a horizontal distance of 2363 m, which covers the project area, and since the depression angles encountered during the project were 30° or greater, 1 cm represents an upper limit to the error resulting from our assumption of linear ray paths from the surface to the bottom, and a correction for refraction is unnecessary.

TABLE III
The Error Comparation at Different Angles

Depth : 1400 meters

Depression Angle (degree)	Horizontal Distance (m)	Ray Trace (m)	Straight Line (m)	Error (cm)
0	13429.23	13508.3850	13502.0064	637.86
10	6637.03	6783.2984	6783.0795	21.89
20	3647.10	3906.6140	3906.5806	3.34
30	2363.42	2746.9658	2746.9562	0.96
60	801.28	1613.0859	1613.0853	0.06
90	0.0	1400.0	1400.0	0.0

IV. RELATIVE POSITION DETERMINATION OF THE TRANSPONDER ARRAY

A. INTRODUCTION

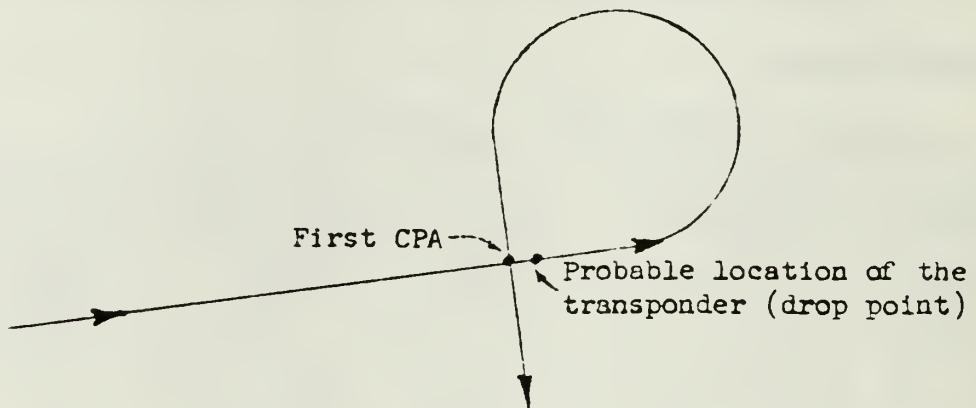
Two methods to determine relative positions are discussed in this chapter. They are basic baseline crossing and the modified baseline crossing method; the former is described by Hart (1967) and Haehnle (1967), the latter is described by Fubara and Mourad (1972). The main difference between the methods is that Hart and Haehnle require accurate transponder depths, but Fubara and Mourad do not.

The cloverleaf method used to determine transponder depths is described by Haehnle (1967) is introduced in the next section.

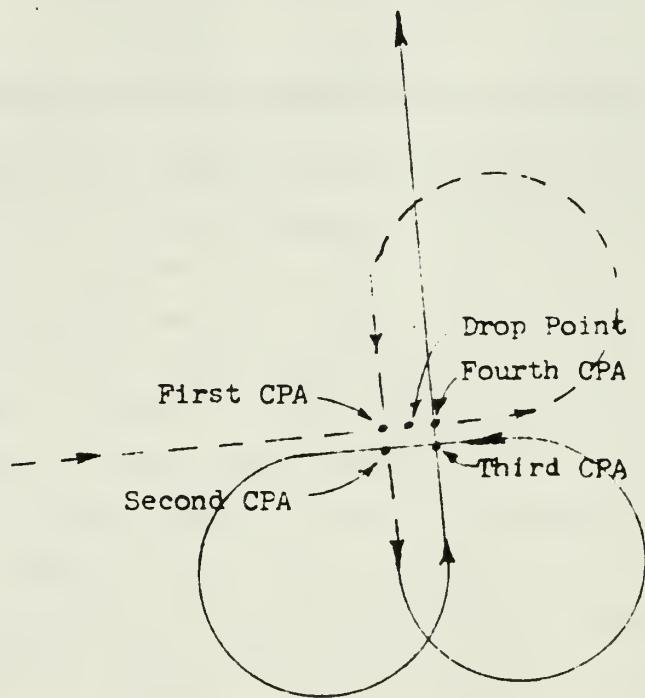
B. CLOVERLEAF METHOD FOR TRANSPONDER DEPTH DETERMINATION

An initial approximation to the depths of bottom-mounted acoustic transponders can be obtained from a knowledge of the water depth at the point where the mooring is released from the ship. However, the unit may move laterally while descending, or bottom topography may have considerable relief not evident on the echo sounder record. Hence, a more accurate method of depth determination is required. Here the depth is defined as the distance between the hull-mounted transducer and the bottom-anchored transponder when the vessel is positioned exactly over the transponder.

The ship need not be directly above the transponder but should be in close proximity to it. Although it is very difficult to cross over the transponder exactly with the ship, the depth may be approximated as the minimum slant range when the transponder is crossed over many times using a "cloverleaf" method (Figure 4.1). Thus, if a transponder



a. The first pass through the transponder location.



b. Maneuver for obtaining four CPA's.

Figure 4.1 Determination of Transponder Depth.

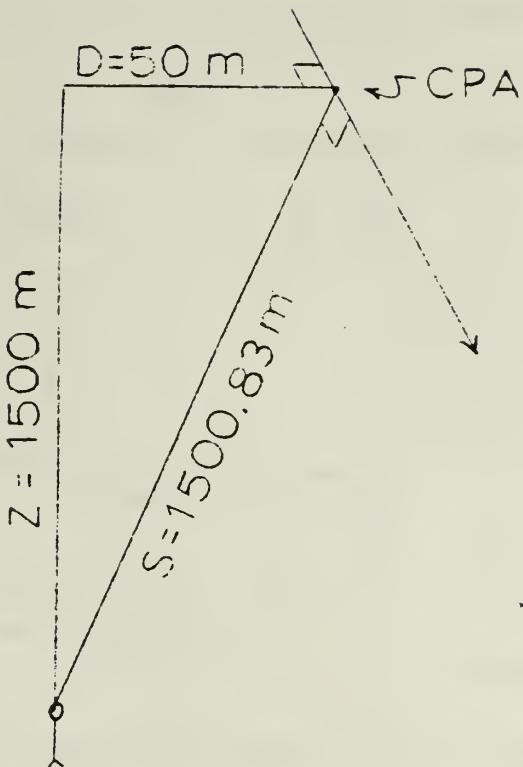


Figure 4.2 Example of Depth Difference.

is at a depth of 1500 m and the ship is displaced 50 meters horizontally from it, the depth determined by the cloverleaf method would be in error by only 0.83 meters (Figure 4.2).

The procedure used for running a cloverleaf maneuver is as follows (Haehnle, 1967):

1. Each time a transponder is dropped from the ship the ship's position is noted.
2. The ship proceeds toward the transponder drop point and passes near it; A closest point of approach (CPA) is then determined and recorded.
3. The ship then turns, crossing the first track perpendicularly and passing near the first CPA. Once again, the CPA of this run is recorded.
4. The above maneuver is repeated until at least four such CPA's have been obtained, each time attempting to pass through the previous CPA perpendicular to the track.

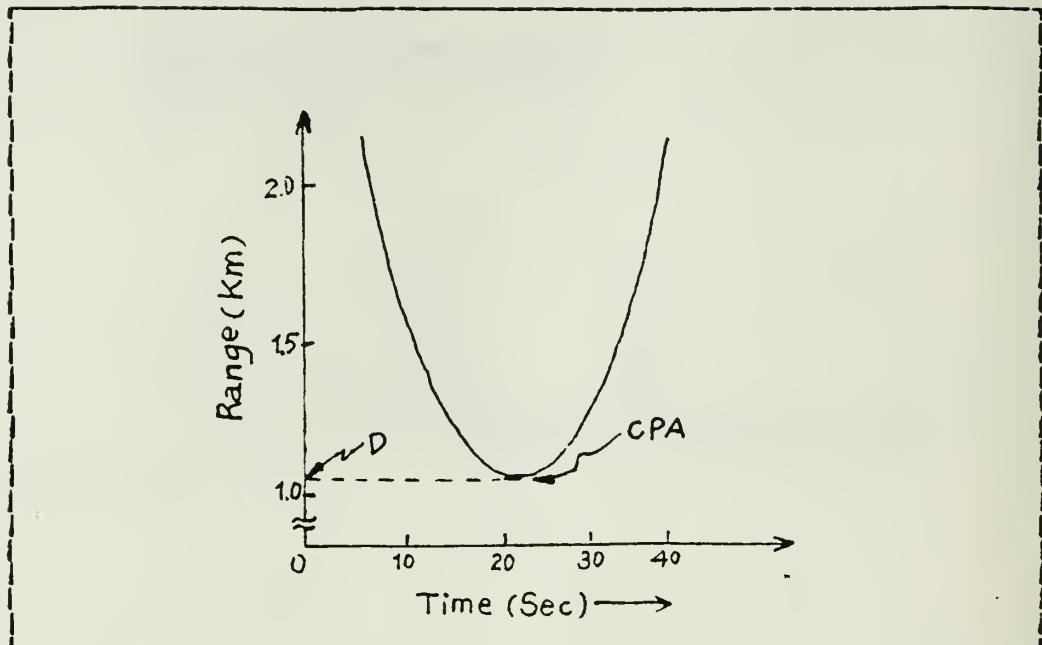


Figure 4.3 Sample of Depth Determination.

5. The depth of the transponder is determined by plotting the slant ranges versus time for each of the four CPA runs. The minimum point of this curve is considered as the minimum slant range for that particular transponder (Figure 4.3).

The R/V ACANIA ran similar cloverleaf patterns on 21 May 1985. She crossed near each transponder many times as closely as possible. The minimum distances and corrected depths are listed in Table IV .

TABLE IV
Transponder Depth by Cloverleaf Maneuver
(From Oceano [1985, p.12])

Transponder	Corrected Depth (m)
1	1366.9
2	1331.3
3	1381.5
4	1406.7

C. BASELINE CROSSING METHODS

Baseline length can be determined utilizing the baseline crossing method described by Haehnle (1967) and Hart (1967), which involves repeatedly transiting the baseline while simultaneously measuring ranges to two transponders. The minimum sum of the two horizontal ranges as determined from measured slant ranges and known transponder depths is taken to be the baseline length.

The basic baseline crossing method requires a knowledge of:

1. The ship's course and speed,
2. The acoustic slant range (or the two-way travel times from the ship to two adjoining ocean-bottom transponders, and
3. The harmonic mean sound velocity.

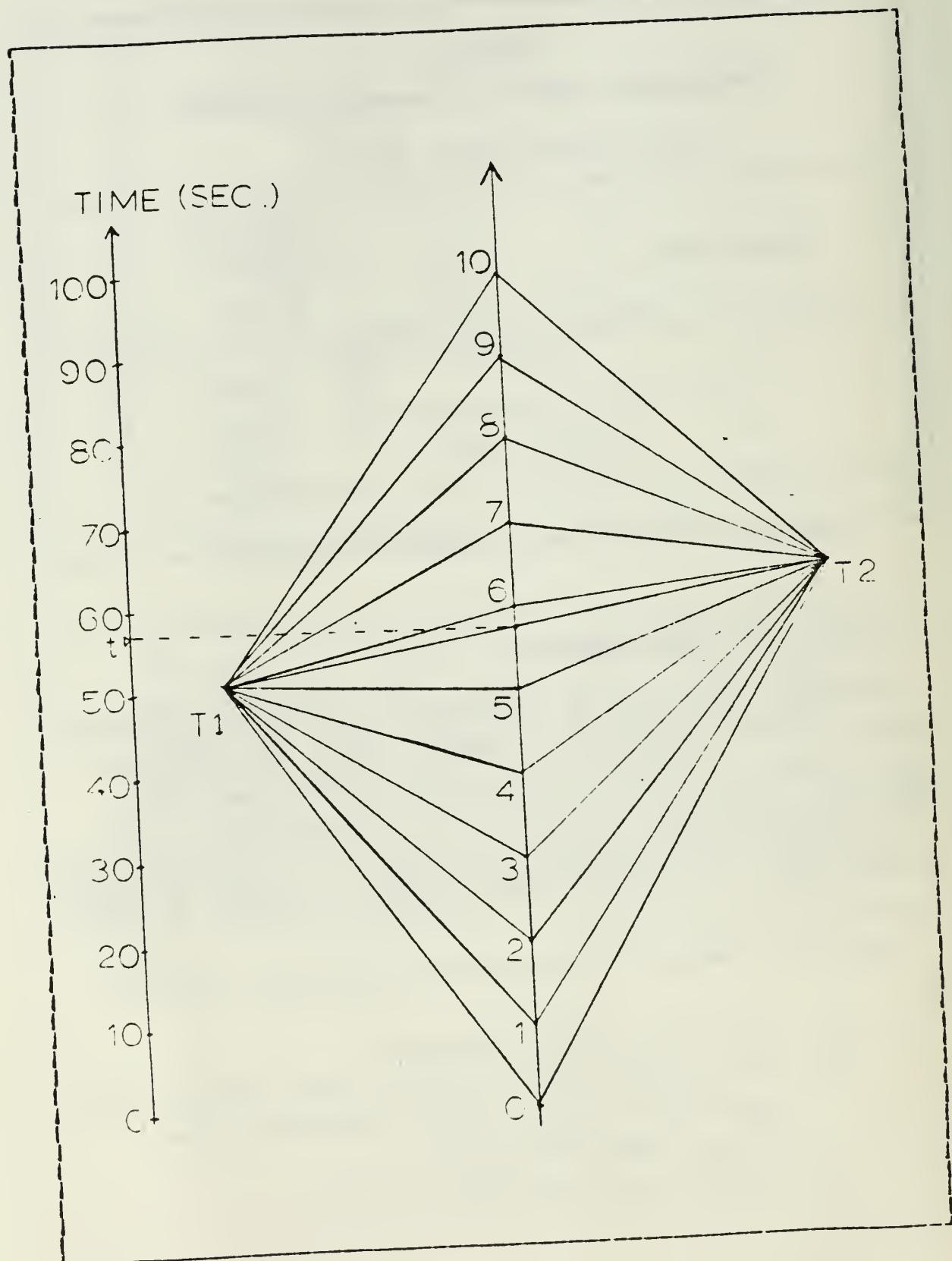


Figure 4.4 Sample of the Baseline Crossing-

The ship, travelling at a constant speed and heading, crosses the lines joining the transponders as shown in Figure 4.4. The acoustic slant ranges are recorded as functions of time during the crossings. The sum of two horizontal ranges (the distance T1-T2 in Figure 4.4) is minimum when the ship is in the vertical plane containing the two transponders. Similarly, for a four-transponder array, the distances T1-T2, T2-T3, T3-T4, T1-T4, T1-T3 and T2-T4 are determined. The minimum sums can be obtained by least-squares fitting the horizontal ranges against time and obtaining the vertex of the parabola as shown in Figure 4.5. During the experiment four to six crossings were made for each baseline between transponders. The results are discussed below in Sections 1 and 2.

Although the baseline crossing method is relatively simple with respect to data reduction, it is operationally time consuming. During the experiment it took one hour and thirty minutes for each baseline crossing. Thus the eight crossings (4 for north-south crossing, 4 for east-west crossing) took a total of 12 hours.

1. Basic Baseline Crossing Model

Figure 4.4 shows a ship track crossing a baseline. The ship's transducer was set to interrogate the bottom acoustic transponders every 10 seconds for this experiment. Ranges were obtained from each transponder. It is easily seen (Figure 4.4) that, as the ship approaches the baseline T1T2, the total horizontal range at the first ship point (0) is greater than at the second ship point (1), and so on, until a minimum sum is reached at the instant of baseline crossing; from there on the total increases for each point on the track. Since the ship maintains a constant course and speed, the mathematical function which describes this relationship is a parabola (Figure 4.5) of the form (Hart, 1967):

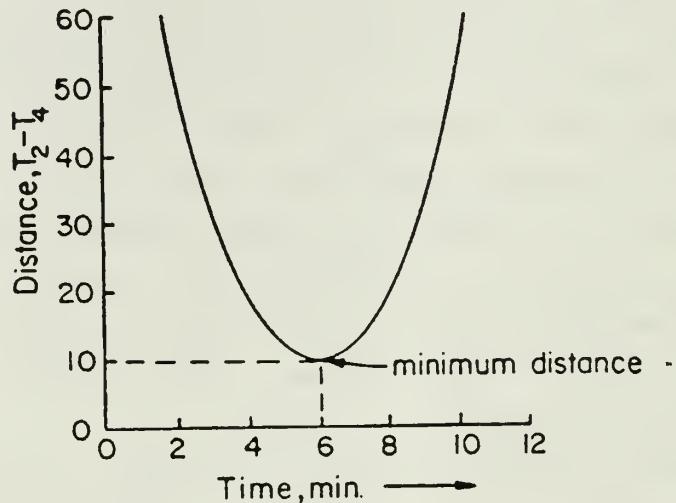


Figure 4.5 Sample of Range-Time Parabola Showing Baseline Crossing Time.

$$R = A T^2 + B T + C$$

(4.1)

where

A, B, C are constants to be determined,

T is time [s], and

R is total horizontal range between transponders
[m].

To determine the unknown variables A, B and C in Equation (4.1) a least squares adjustment is performed. At least four sets of data $R = R(T)$ are needed to solve Equation (4.1). Since observed parameters contain random error, Equation (4.1) can be rewritten as:

$$A T^2 + B T + C = R + V$$

(4.2)

where

V is the residual of the observation.

In matrix notation this corresponds to:

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ \vdots \\ V_n \end{bmatrix} + \begin{bmatrix} -T_1^2, -T_1, -1 \\ -T_2^2, -T_2, -1 \\ \vdots \\ \vdots \\ \vdots \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ \vdots \\ R_n \end{bmatrix} \quad (4.3)$$

which may be written as:

$$V_{n \times 1} + T_{n \times 3} X_{3 \times 1} = R_{n \times 1} \quad (4.4)$$

where V_i are the residuals of the observations,

T_{ij} are the numerical coefficients of the unknown variable,

X_j are the unknown variables (A, B, C), and

R_i are constant terms of total horizontal range.

The standard deviation of the mean of sound velocity is ± 0.3 m/s (Wilson, 1960). So, the standard deviation of the distance is the following, where time is assumed errorless :

$$\sigma_{R_i} = \sigma_c T \quad (4.5)$$

Each observation is considered to be independent and uncorrelated. Since a measurement of high precision has a small variance, and one of low precision has a large variance, the higher the weight the higher is the precision and vice versa. Accordingly, the weight W of a single obser-

tion is defined as a quantity that is inversely proportional to the variances of the observations (σ^2) (Mikhail, 1981, p.66).

$$w = \frac{\sigma^2}{\sigma_0^2} / \sigma^2 \quad (4.6)$$

where σ_0^2 = variance of unit weight

For the adjustment of Equation (4.4), the least squares criterion requires minimization of the weighted function G (Mikhail, 1981, pp.69-73) :

$$G = w_1 v_1^2 + w_2 v_2^2 + \dots + w_n v_n^2 = \sum_{i=1}^n w_i v_i^2 \quad (4.7)$$

which in matrix form is written as

$$G = V^T W V \quad (4.8)$$

where

$$W = \begin{bmatrix} w_1 & 0 & \dots & 0 \\ 0 & w_2 & & \cdot \\ \cdot & \cdot & \ddots & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & w_n \end{bmatrix}_{n \times n}$$

Now Equation (4.4) can be rearranged as:

$$V = R - T X \quad (4.9)$$

Substituting for V into Equation (4.8), get

$$G = (R - TX)^T W (R - TX) = (R^T - X^T T^T) W (R - TX) \quad (4.10)$$

so,

$$G = R^T W R - X^T T^T W R - R^T W T X + X^T T^T W T X \quad (4.11)$$

Since G is a scalar, the right hand side of Equation (4.10) is also a scalar. Furthermore, the transpose of a scalar is equal to the scalar:

$$X^t T^t W R = (X^t T^t W R)^t = R^t W T X \quad (4.12)$$

and

$$W^t = W \quad (4.13)$$

Thus, Equation (4.10) can be written as:

$$G = R^t W R - 2R^t W T X + X^t (T^t W T) X \quad (4.14)$$

In Equation (4.14) all matrices are constants, except X , the matrix of unknowns. For G to be a minimum, its partial derivative with respect to X must be equal to zero:

$$\frac{\partial G}{\partial X} = -2R^t W T + 2X^t (T^t W T) = 0 \quad (4.15)$$

and,

$$(T^t_{3 \times n} W_{n \times n} T_{n \times 3}) X_{3 \times 1} = T^t_{3 \times n} W_{n \times n} R_{n \times 1} \quad (4.16)$$

giving the solution for X :

$$X = (T^t W T)^{-1} T^t W R \quad (4.17)$$

Now, the minimum distance occurs when $dR/dT = 0$
which is given below:

$$T = -B / 2A \quad (4.18)$$

Thus, to get the minimum distance of the sum, one may substitute the time from Equation (4.18) into Equation (4.1):

$$R \text{ (min.)} = B^2/4A - B^2/2A + C \quad (4.19)$$

where A, B, C are given by Equation (4.17).

The variances of A, B and C (Mikhail, 1981, p258) are:

$$\sigma_x^2 = \sigma_o^2 (T^t W T)^{-1} \quad (4.20)$$

The standard deviation of T in Equation (4.18) is:

$$\sigma_T = [(\sigma_A B/2A^2)^2 + (\sigma_B/2A)^2]^{1/2} \quad (4.21)$$

The standard deviation of the minimum distance R in Equation (4.19) is:

$$\sigma_R = [(T^2 \sigma_A^2)^2 + (T \sigma_B^2)^2 + \sigma_C^2 + (2AT+B)^2 \sigma_T^2]^{1/2} \quad (4.22)$$

2. Data Processing

During the experiment at sea, both north-south and east-west baseline crossings were run (Figures 4.6 and 4.7) using data collected by Oceano Instruments (1985a). For each baseline crossing, 11 consecutive interrogation points were fit to a parabola by least squares. The reason for using 11 points is that this number is sufficient to define the parabola near its minimum. The minimum value of the parabola can be considered to define the baseline length. The parabolas shown in the Figures 4.8, 4.9, 4.10, 4.11,

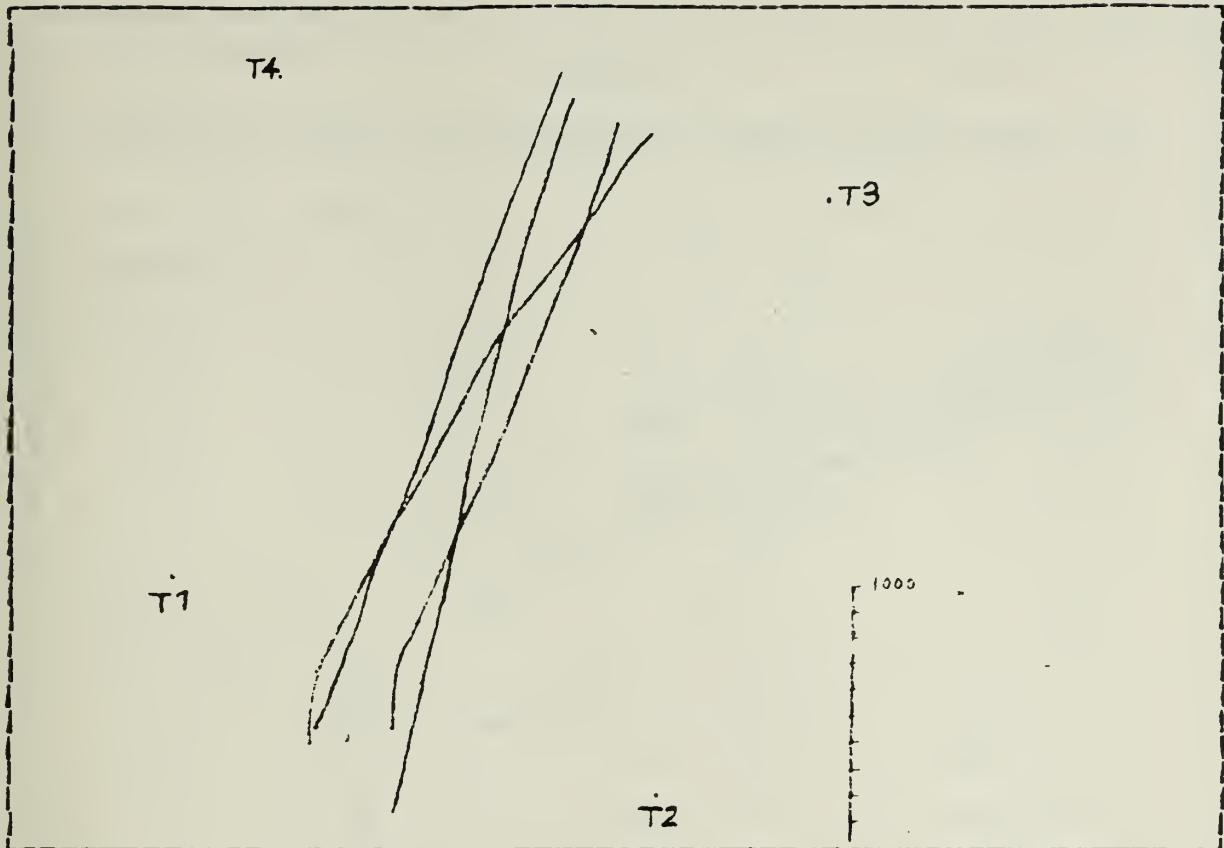


Figure 4.6 North-South Baseline Crossing.

4.12 and 4.13 (at the end of this section) are for single crossings between T1-T2, T2-T3, T3-T4, T1-T4, T1-T3 and T2-T4. The results are tabulated in detail in Table V, which gives the baseline lengths and their standard deviations for each crossing. To get the mean of these lengths, one can use the following equation:

$$\bar{R}_m = (R_1 + R_2 + \dots + R_n) / n = f(R) \quad (4.23)$$

Since the standard deviation of the mean length σ_m in Equation (4.23) is:

$$\sigma_m = [(\delta_1 \partial f / \partial R_1)^2 + (\delta_2 \partial f / \partial R_2)^2 + \dots]^{1/2} \quad (4.24)$$

we have

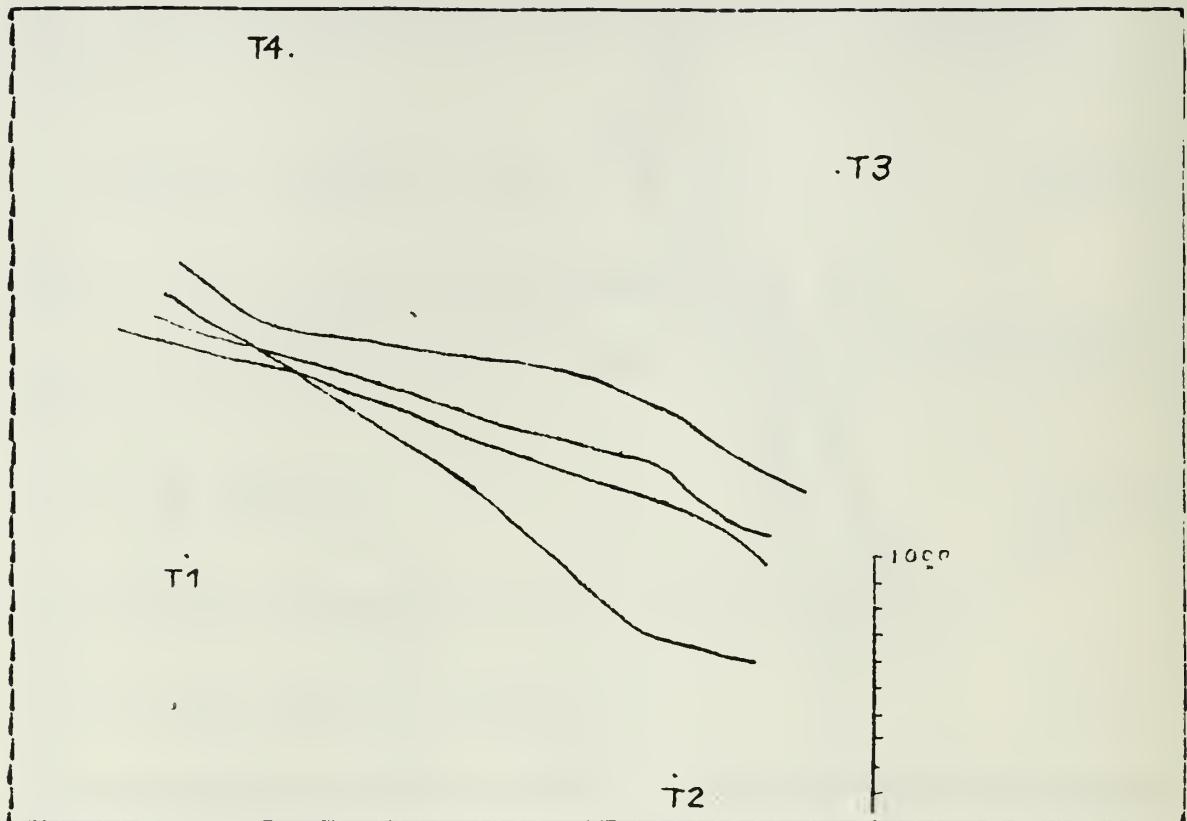


Figure 4.7 East-West Baseline Crossing.

$$\sigma_m = [(\delta_1/n)^2 + (\delta_2/n)^2 + \dots + (\delta_n/n)^2]^{1/2} \quad (4.25)$$

Equations 4.23 and 4.25 are used with the data of Table V to compute the mean of baseline lengths and their standard deviations (Table VI).

Lyman Burke of Oceano Instruments and L. Spielvogel of Seaco, Inc., have also computed the baseline lengths. They use a different method and technique using most of the data collected during the experiment. Their results and the differences between them are tabulated in Table VII. Due to our more limited data set and uncertainty in the depth for the baseline crossing method, these results differ from those obtained with the crossing method by about 2 m.

TABLE V
Baseline Lengths Determined by Baseline Crossings (m)

Crossing	T1 - T2	T2 - T3	T3 - T4
1.	1793.32±2.43	1993.09±0.81	1972.39±2.50
2.	1792.62±2.17	1991.92±2.73	1969.50±2.02
3.	1793.67±2.27	1992.75±2.38	1970.29±2.19
4.	1791.77±2.22	1994.33±2.09	1969.97±2.23
Crossing	T1 - T4	T1 - T3	T2 - T4
1.	1860.81±3.25	2789.62±2.20	2588.66±2.03
2.	1858.43±1.81	2789.77±1.62	2589.70±1.94
3.	1858.63±2.23	2789.45±2.11	2587.88±1.14
4.	1858.48±2.08	2789.73±1.63	2588.17±2.21
5.	1858.65±2.04	2790.22±2.26	2589.22±2.26
6.			2588.82±2.20

3. Modified Baseline Crossing Method

Mourad and Fubara (1972) developed this method. Their method determines the relative positions of ocean-bottom transponders and the depth of each transponder. There is no necessity to accurately measure each depth directly; this is the main difference from the previous work. The physical principle of this method is shown in Figure 4.14. S is the baseline length and L1 and L2 are a pair of coplanar ranges from the ship to transponders 1 and 2 whose depths are Z1 and Z2, respectively. The mathematical expression for this configuration is:

TABLE VI
The Mean of The Baseline Lengths

	Mean Value (m)		Standard Deviation (m)
T1 - T2	1792.84		1.14
T2 - T3	1992.42		0.93
T3 - T4	1970.54		1.12
T1 - T4	1859.00		1.04
T1 - T3	2789.76		0.89
T2 - T4	2588.74		0.82

$$S = (L_1^2 - Z_1^2)^{1/2} + (L_2^2 - Z_2^2)^{1/2} \quad (4.26)$$

To determine the three unknowns S , Z_1 and Z_2 by least squares method, at least 4 pairs of L_1 and L_2 are needed. Now, Equation (4.26) can be rewritten as follows:

$$F = S - (L_1^2 - Z_1^2)^{1/2} - (L_2^2 - Z_2^2)^{1/2} \quad (4.27)$$

Assume

$$A_1 = (L_1^2 - Z_1^2)^{1/2} \quad (4.28)$$

$$A_2 = (L_2^2 - Z_2^2)^{1/2} \quad (4.29)$$

Equation (4.27) is linearized and partially differentiated:

TABLE VII
Baseline Length Determination by Oceano
Instruments and L. Spielvogel

Lines	Distance (m)		Difference (m)
	Oceano	Spielvogel	
T1 - T2	1792.68	1791.7	0.98
T2 - T3	1997.46	1995.9	1.56
T3 - T4	1973.92	1972.6	1.32
T4 - T1	1861.86	1860.5	1.36
T1 - T3	2792.69	2790.8	1.89
T2 - T4	2592.18	2590.3	1.88

$$\frac{\partial F}{\partial L_1} = -A_1^{-1}L_1 \quad (4.30)$$

$$\frac{\partial F}{\partial L_2} = -A_2^{-1}L_2 \quad (4.31)$$

$$\frac{\partial F}{\partial S} = 1 \quad (4.32)$$

$$\frac{\partial F}{\partial Z_1} = A_1^{-1}Z_1 \quad (4.33)$$

$$\frac{\partial F}{\partial Z_2} = A_2^{-1}Z_2 \quad (4.34)$$

T1-T2 BASELINE CROSSING

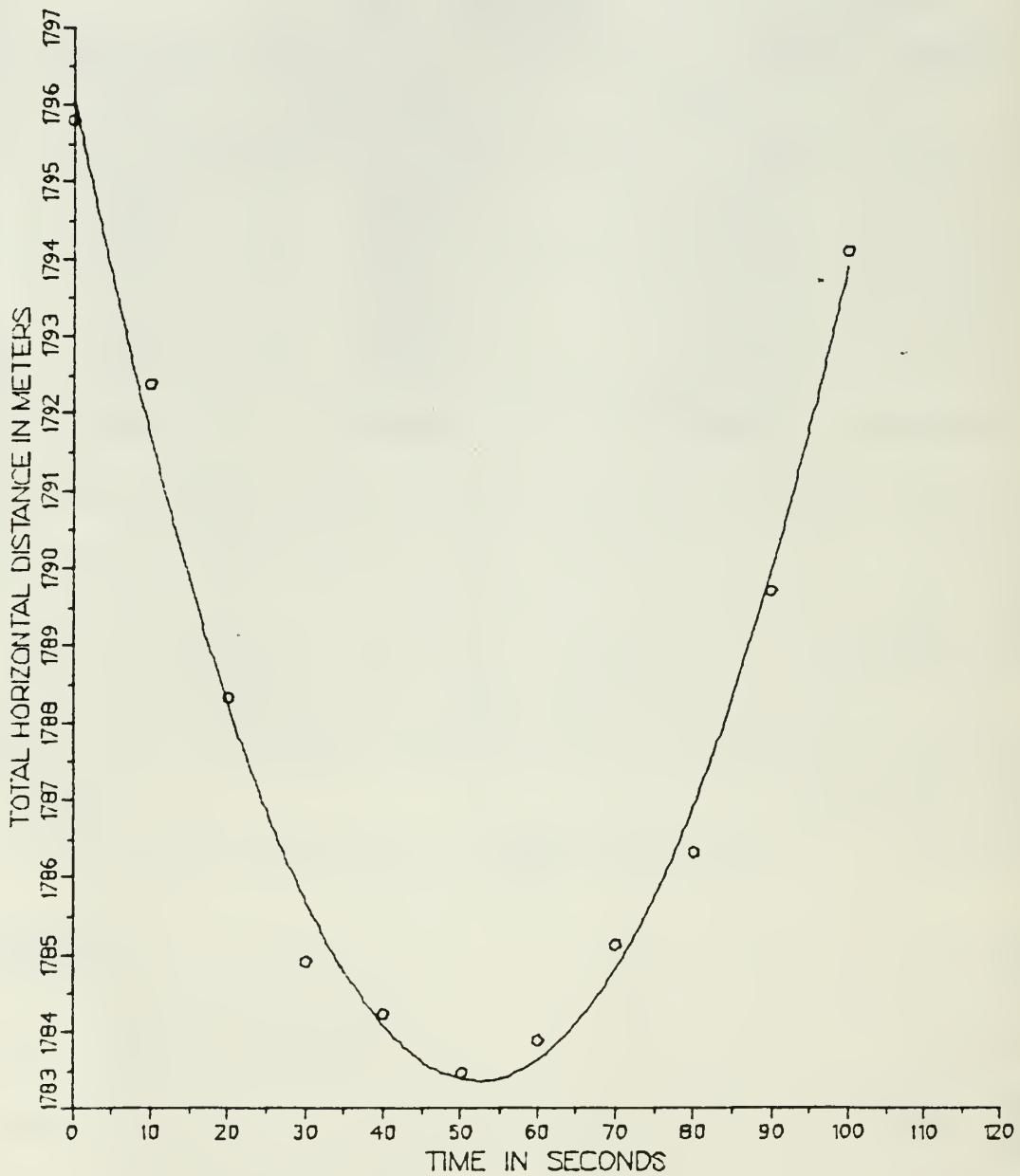


Figure 4.8 The Best Fit of Parabola for T1-T2 Crossing.

T2-T3 BASELINE CROSSING

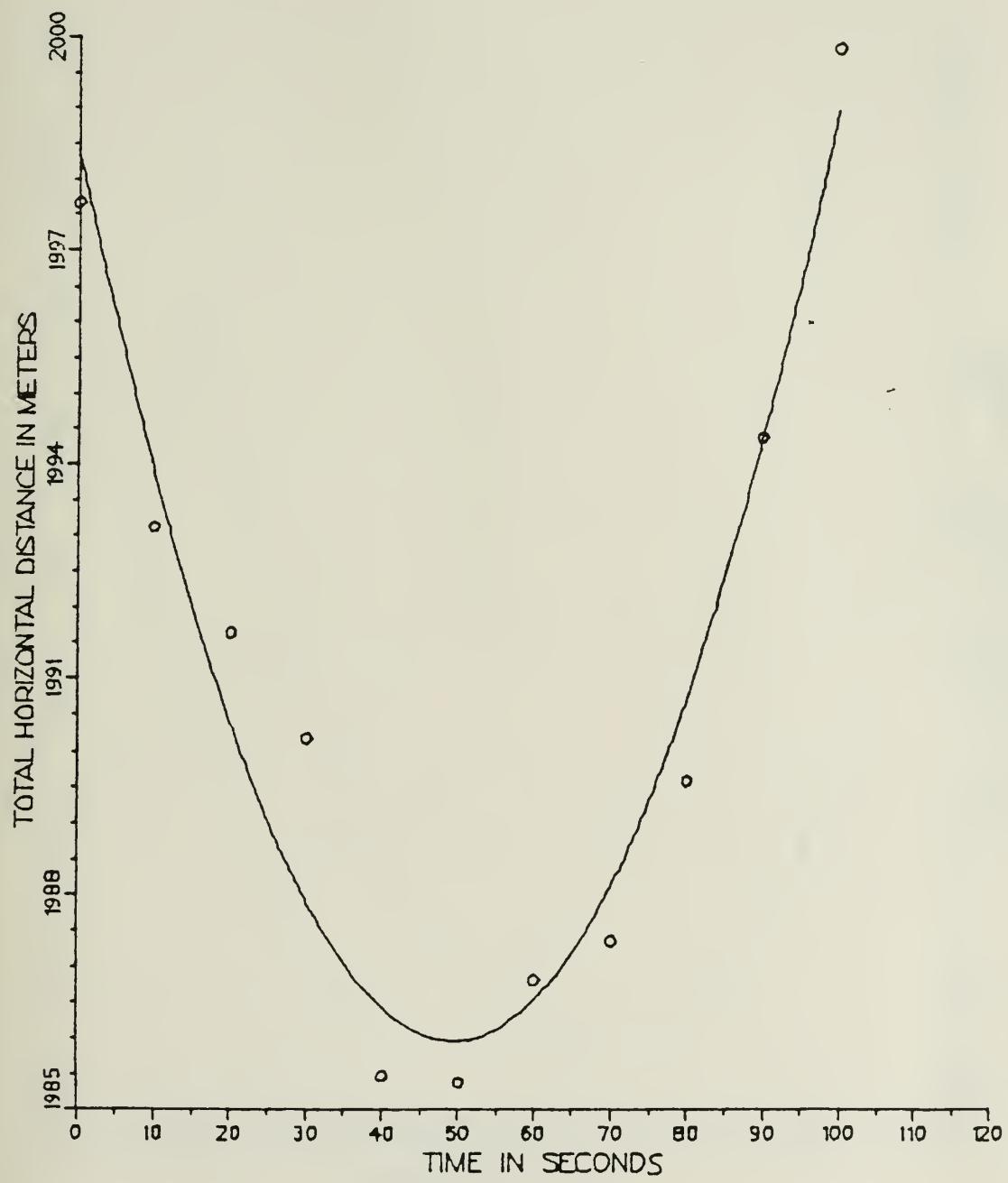


Figure 4.9 The Best Fit of Parabola for T2-T3 Crossing.

T3-T4 BASELINE CROSSING

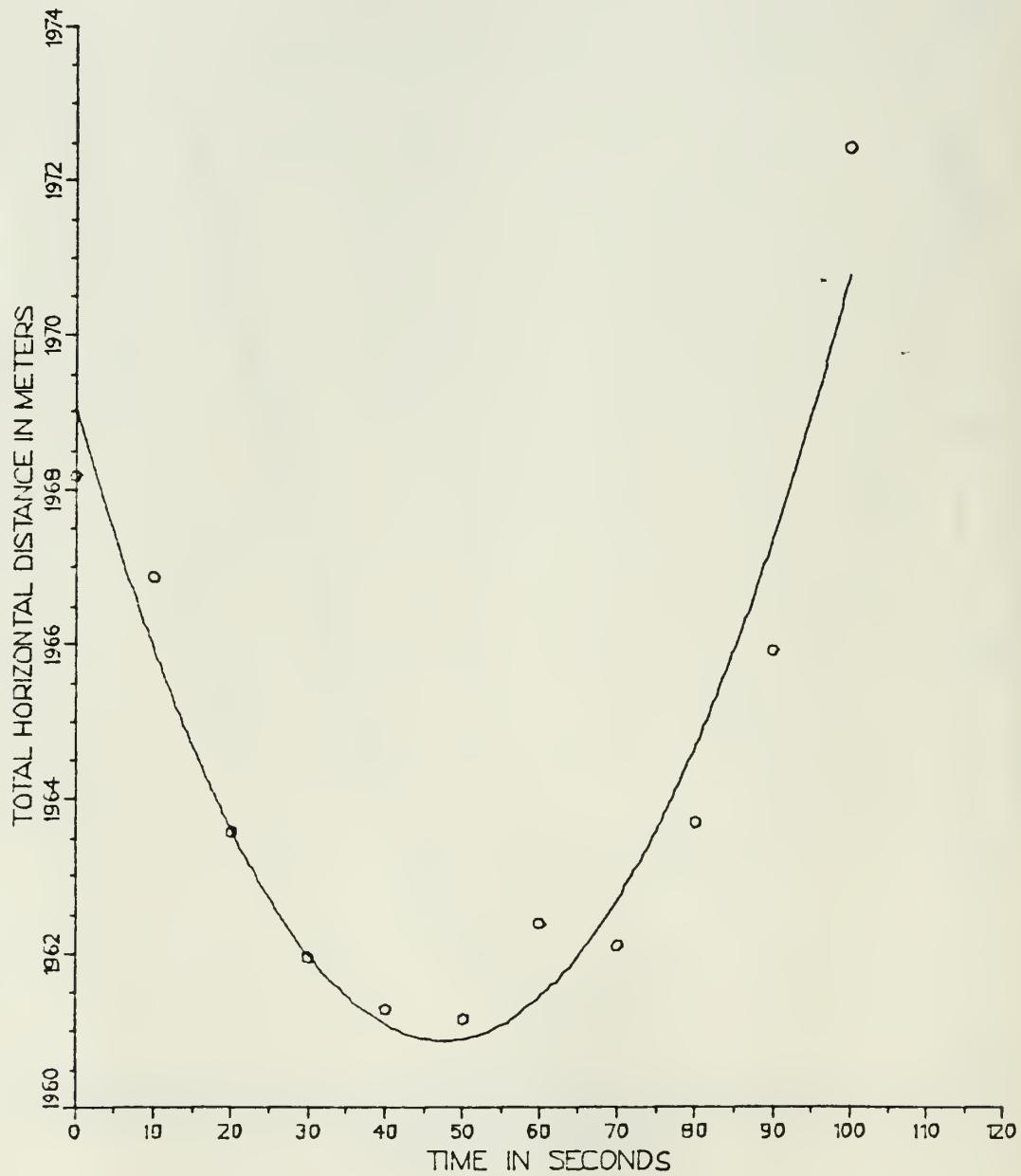


Figure 4.10 The Best Fit of Parabola for T3-T4 Crossing.

T1-T4 BASELINE CROSSING

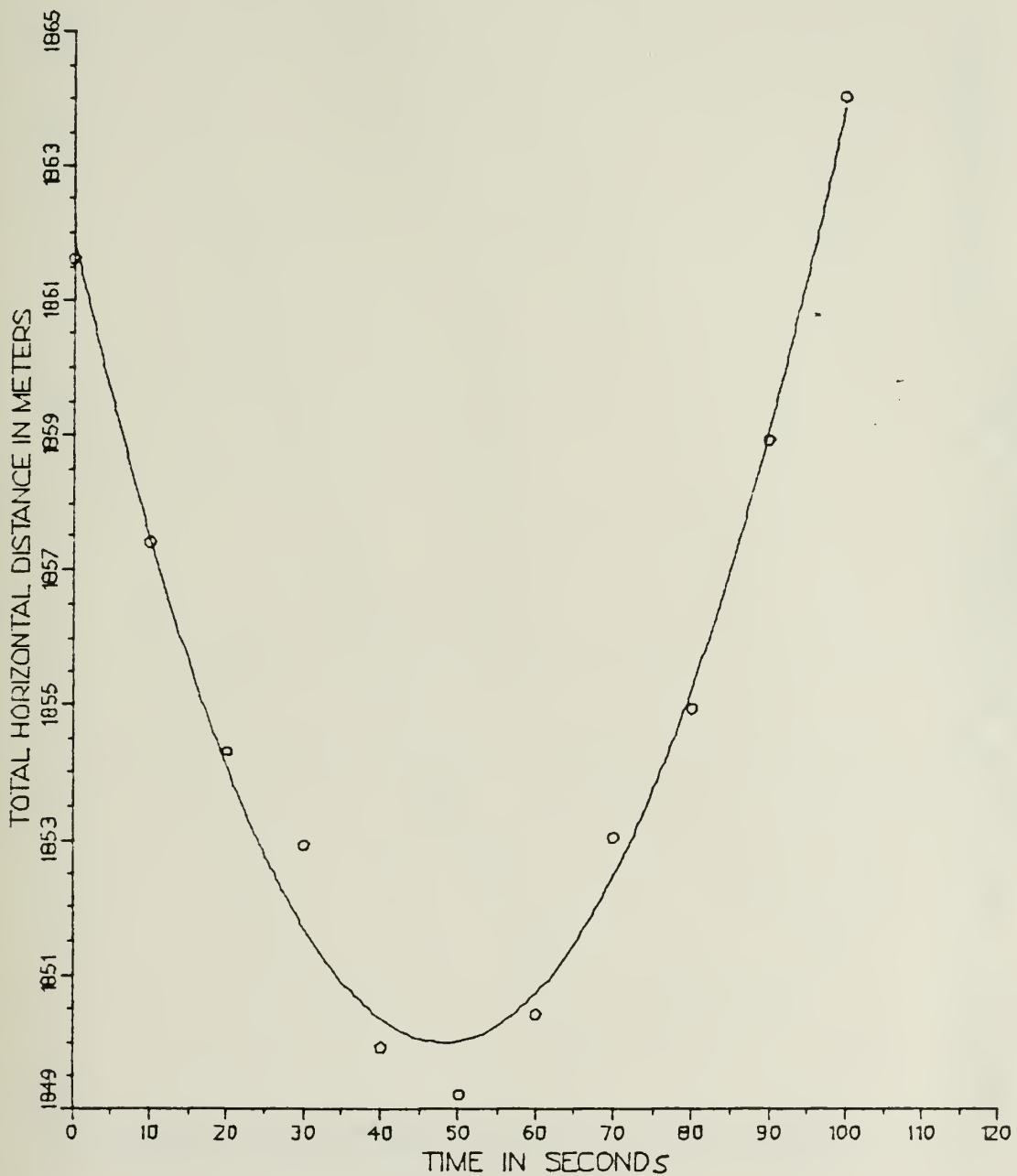


Figure 4.11 The Best Fit of Parabola for T1-T4 Crossing.

T1-T3 BASELINE CROSSING

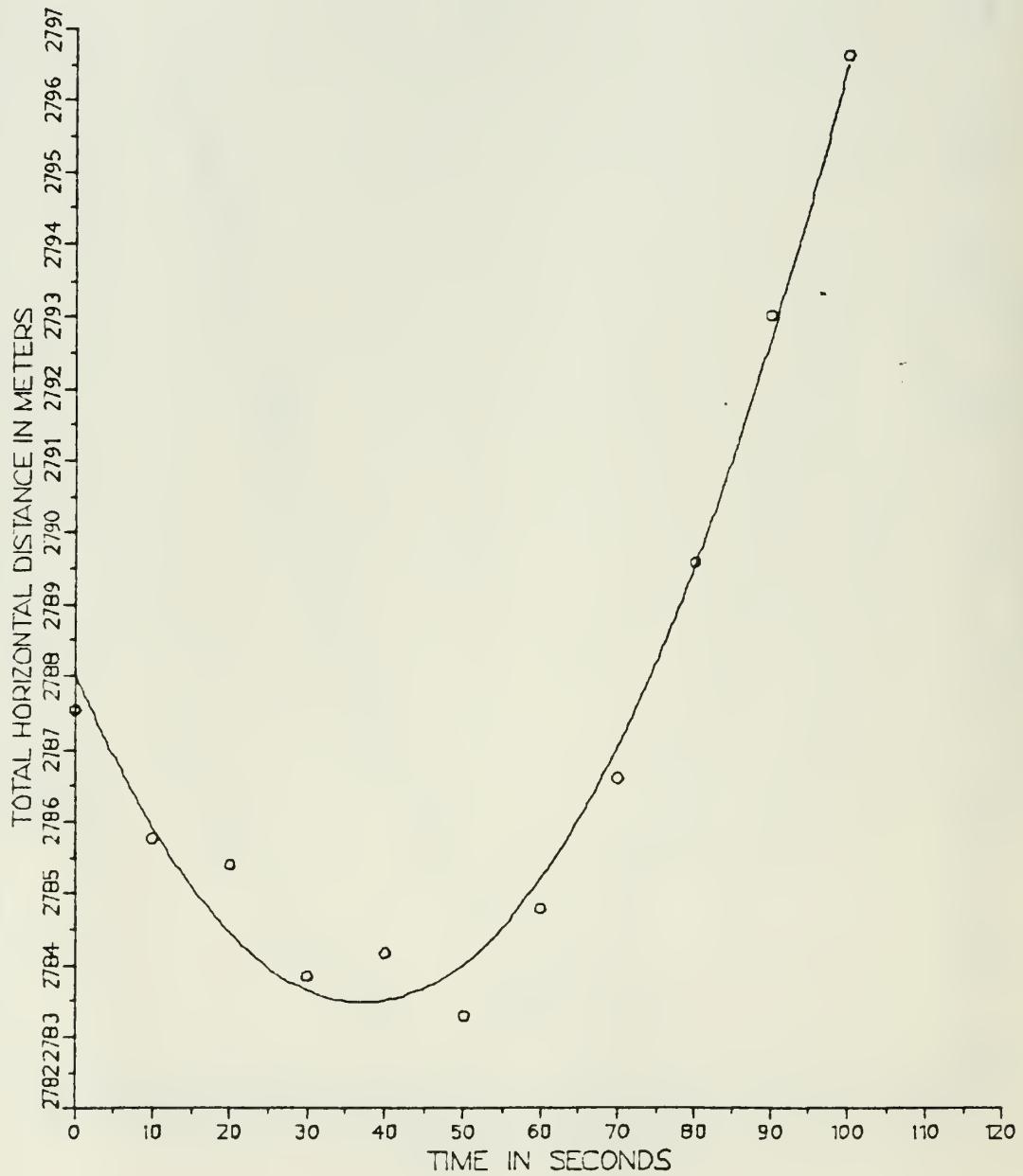


Figure 4.12 The Best Fit of Parabola for T1-T3 Crossing.

T2-T4 BASELINE CROSSING

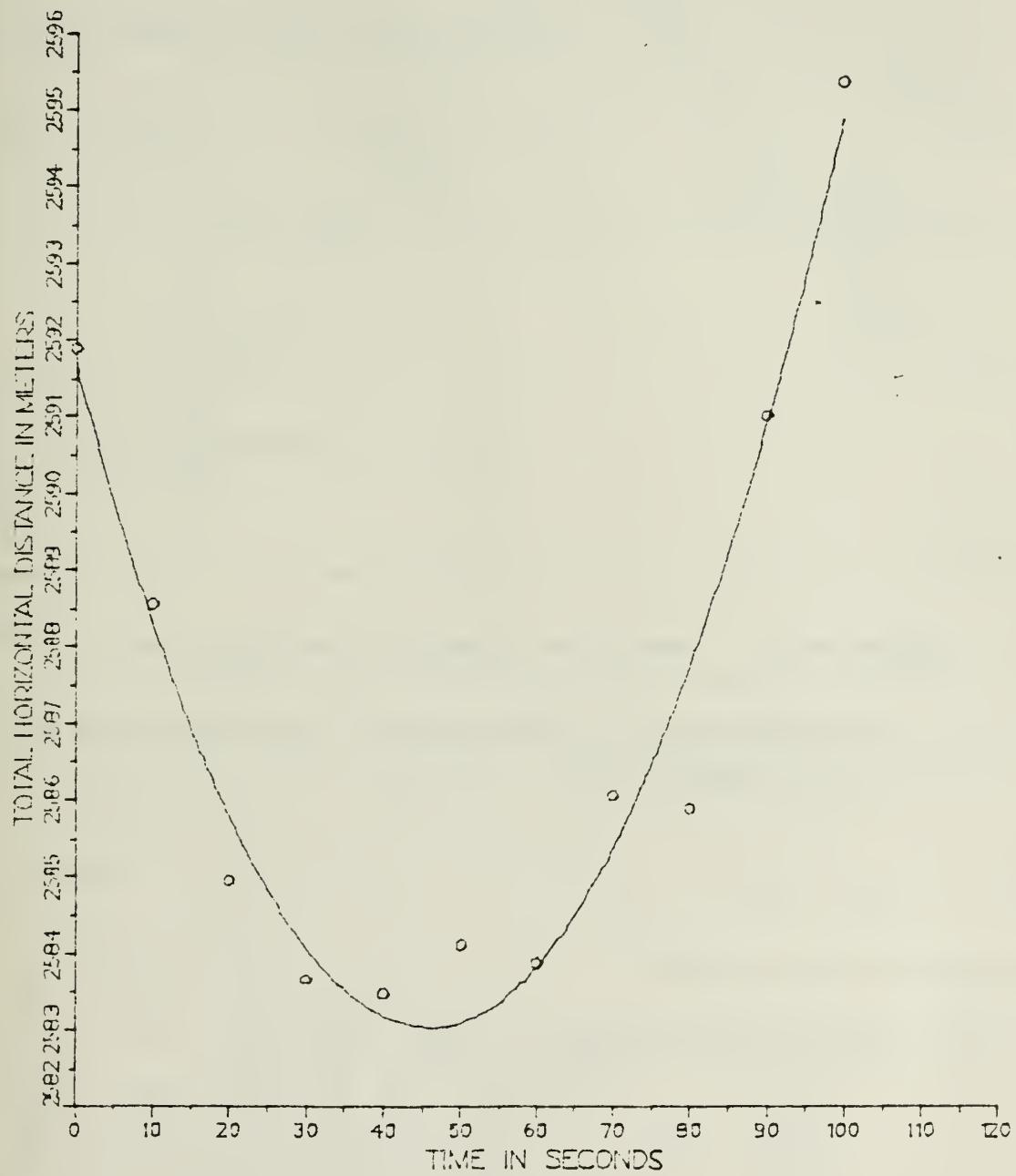


Figure 4.13 The Best Fit of Parabola for T2-T4 Crossing.

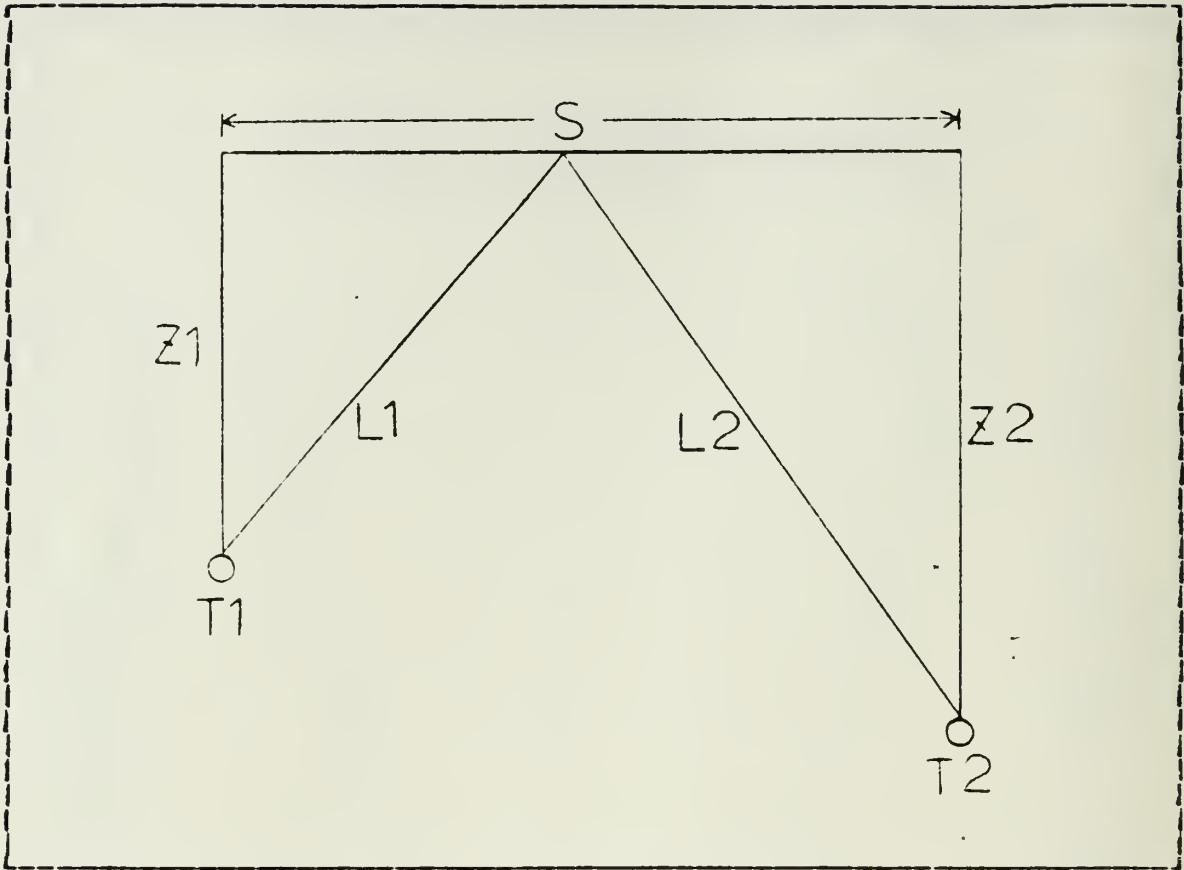


Figure 4.14 Baseline Crossing Configuration.

The resultant observation equation for a least-squares solution is (Mourad , 1972, p.21):

$$A \Delta + B V + W = 0 \quad (4.35)$$

which can be written as:

$$\begin{pmatrix} 1, A_1^{-1}Z_1, A_2^{-1}Z_2 \\ \cdot & \cdot & \cdot \end{pmatrix} \begin{pmatrix} \Delta S \\ \Delta Z_1 \\ \Delta Z_2 \end{pmatrix} + \begin{pmatrix} -A_1^{-1}L_1, -A_2^{-1}L_2, 0 \\ 0, 0, -A_1^{-1}L_2 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ \vdots \\ \vdots \end{pmatrix} + \begin{pmatrix} W_1 \\ W_2 \\ \vdots \\ \vdots \\ W_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{pmatrix} \quad (4.36)$$

Thus, the least-squares solution (Mourad, 1972, p.21) gives:

$$\Delta = -[P_x + A^t (B P_i^{-1} B^t)^{-1} A]^{-1} A^t (B P_i^{-1} B^t)^{-1} W \quad (4.37)$$

$$v = -P_i^{-1} B^t (B P_i^{-1} B^t)^{-1} (A \Delta + W) \quad (4.38)$$

The variance of unit weight σ_0 is:

$$\sigma_0 = \{[(B P_i^{-1} B^t)^{-1} (A \Delta + W)]^t W / df\}^{1/2} \quad (4.39)$$

The variance-covariance matrix for the adjusted parameters X_a is:

$$\sigma_0 [P_x + A^t (B P_i^{-1} B^t)^{-1} A]^{-1} \quad (4.40)$$

The adjusted parameters X_a is:

$$X_a = X_o + \Delta \quad (4.41)$$

where

- A is a coefficient matrix for the unknowns correction,
- B is a coefficient matrix for the observation,
- v is a vector of residuals, representing the corrections to observed ranges, L1 and L2,
- Δ is the correction to assumed X_o ,
- W is a constant matrix in Equation (4.27), when using assumed value X_o ($S_o, L1_o, L2_o$),
- P_i is the weight matrix of L_o ,
- P_x is the weighting function associated with X_o ,
- σ_0 is the variance of unit weight
- X_a is true value of S, Z1 and Z2, and
- X_o is an approximate value of X_a .

The weight matrices are taken to be unity as the measurements were of equal precision. Table VIII shows the

resulting horizontal distances between transponders and the depths of each transponder. These results are obtained by measuring pairs of slant ranges at the instant the ship crosses the baseline. Since the ship crossed the baseline four times for the baseline crossing method, there are only four pairs of slant ranges which can be used; these represent the smallest data sets needed for a least squares solution.

The baseline lengths computed by Oceano Instruments and Dr. Spielvogel are given in Table IX. The difference between Oceano's and Spielvogel's computed values are within the precision of the system. My results in Table VIII differ from Table IX, since Oceano and Spielvogel used much larger data sets (more than 250 data points), whereas I used the much smaller sets (11 points for each of four crossings) required by the two baseline crossing methods, namely eleven data points for each of four crossings. Since Oceano and Spielvogel used a resection method with four transponders, it is not necessary for the ship to travel the baseline repeatedly: The only requirement is that the ship be in the vicinity of the center of the experimental area, where data can be collected continuously. On the contrary, the crossing methods require a large amount of ship time to collect a small amount of data. Thus, it is not surprising that there should be good agreement between Oceano and Spielvogel and rather poorer agreement between my results and theirs.

TABLE VIII
Results of Modified Baseline Crossing Method

Trans- ponder Pairs	Horizontal Distance Between Transponders (m)		Transponder Number	Mean Depths (m)
1-2	1782.91±9.51		1	1361.52±4.53
2-3	1993.02±6.30		2	1333.94±2.54
3-4	1964.34±8.32		3	1375.76±5.21
1-4	1836.65±9.21		4	1407.08±3.47
1-3	2815.47±15.60			
2-4	2574.13±7.55			

TABLE IX
Comparison of Results for Different Methods

Lines	Oceano	Spielvogel	Basic	Modified
			Baseline	Baseline
			Crossing	Crossing
T1-T2	1792.68	1791.7	1792.84±1.14	1782.91±9.51
T2-T3	1997.46	1995.9	1992.42±0.93	1993.02±6.30
T3-T4	1973.92	1972.6	1970.54±1.12	1964.34±8.32
T4-T1	1861.86	1860.5	1859.00±1.04	1836.65±9.21
T1-T3	2792.69	2790.8	2789.76±0.89	2815.47±15.6
T2-T4	2592.18	2590.3	2588.74±0.82	2574.13±7.55

V. CONCLUSIONS AND RECOMMENDATIONS

Two methods were used to determine the horizontal distances between the elements of an array of four acoustic transponders lying on the ocean bottom at an average depth of 1370 meters, namely basic and modified baseline crossing methods. The principal difference between the two techniques is that the former requires precise depth information, whereas the latter does not. A "cloverleaf" maneuver was described which may be used to determine the depths of the transponders. Because a larger number of acoustic range observations was used for the basic method, the standard deviations for the horizontal distances between transponders using that technique were about one-tenth those found using the modified method. Our results for the basic method are consistent with independent calculations made by Spielvogel, who used a much larger data set. The differences between our results using the modified method and Spielvogel's are within our relatively large standard deviations, with the exception of the distance measured between transponders 1 and 3. This discrepancy is probably due to the very oblique crossings of the 1-3 baseline which were used.

In addition to the smallness of the data sets used for our calculations other sources of error in our results may stem from the variability of the ship's speed and its pitch and roll, which we did not take into account.

If the absolute positions of the bottom transponders are required, the position of the ship may be determined by an absolute positioning system such as GPS while measuring slant ranges from the ship to each transponder. When at least three or more absolute positions of the ship's acoustic transducer are known, the transponder coordinates

may then be adjusted by the method of least squares to obtain a best fit between measured and calculated slant ranges from the known transducer positions. This method is introduced in the Appendix.

APPENDIX A
ABSOLUTE POSITION OF OCEAN BOTTOM TRANSPONDERS

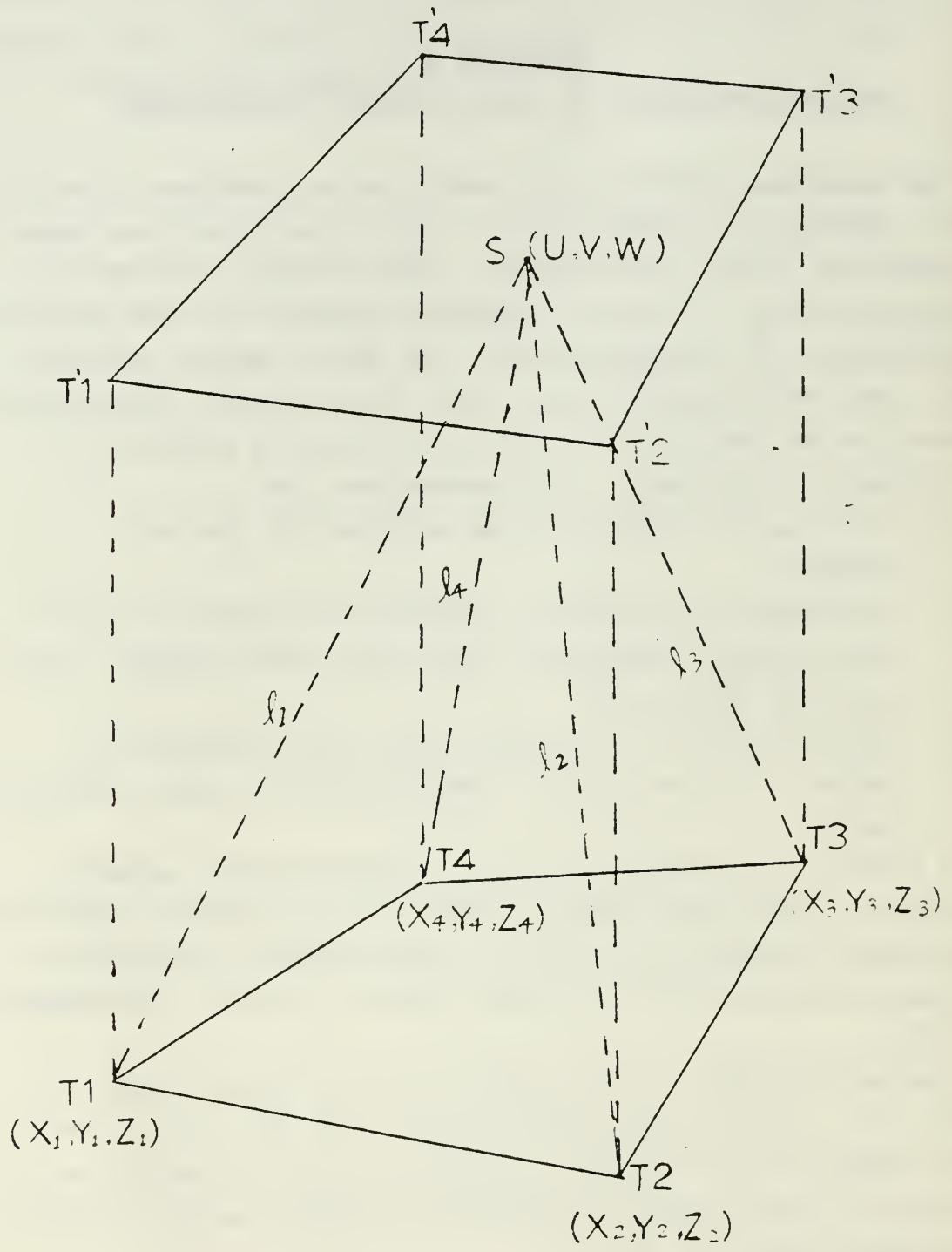
The absolute position of a ship can be obtained from the Global Positioning System (GPS). Here, ship position means the position of the GPS antenna. The antenna position is a function of time. Once the absolute position of the antenna is obtained, and simultaneously, the ship's speed, heading, pitch, roll and acoustic data, then the absolute position of antenna can be used to calculate the absolute position of the bottom transponders. The procedures are:

1. To determine the absolute position of the GPS antenna;
2. To convert the absolute position of antenna to the hull-mounted transducer using the ship heading, pitch and roll data; and
3. Simultaneously, to convert the ship transducer's position to the bottom transponders by using acoustic data.

A method is introduced here to determine the absolute position of sea-floor transponders, if the absolute position of surface transducer is given. Least-squares adjustments must also be supplied to get the best fit of the transponder position.

The three-dimensional geometry used is illustrated in Figure A.1. There are 4 transponders (T_1, T_2, T_3 and T_4). Their estimated coordinates are (X_i, Y_i, Z_i) , the absolute position of ship-mounted transducer is $S(U_j, V_j, W_j)$, and the four slant ranges are ℓ_i .

The adjusted distances $\hat{\ell}_i$ between transducer and transponders can be computed given coordinates of any two points:



**Figure A.1 The Relationship of Transducer (S)
To Transponders, (T_i) ..**

$$\hat{d}_1 = [(U-X_1)^2 + (V-Y_1)^2 + (W-Z_1)^2]^{1/2} \quad (A.1)$$

$$\hat{d}_2 = [(U-X_2)^2 + (V-Y_2)^2 + (W-Z_2)^2]^{1/2} \quad (A.2)$$

$$\hat{d}_3 = [(U-X_3)^2 + (V-Y_3)^2 + (W-Z_3)^2]^{1/2} \quad (A.3)$$

$$\hat{d}_4 = [(U-X_4)^2 + (V-Y_4)^2 + (W-Z_4)^2]^{1/2} \quad (A.4)$$

which can be written simply as follows:

$$\hat{d}_i = [(U_i - X_i)^2 + (V_i - Y_i)^2 + (W_i - Z_i)^2]^{1/2} \quad (A.5)$$

where i are 1,2,3,4. index corresponding to each transponder;

j are 1,2 ••• N. transducer positions at the surface;

U, V, W are coordinate of transducer;

X, Y, Z are coordinate of transponder; and

\hat{d}_i are adjusted distance of slant range.

It was necessary to determine the minimum number of positions (N) of the hull-mounted transducer that were needed to fix the position of four transponders in the three-dimensional coordinate system. There are three unknown variables to be solved for each transducer position, and there are four transducer positions to solve for. To solve for these twelve unknown variables and apply least-squares adjustment, the number of equations must be greater than or equal to the number of unknown variables. This is given by:

$$4N \geq 12$$

(A.6)

where 4 is four equations for each interrogation;
N is number of interrogation; and
12 is twelve unknown for 4 transponders coordinate.

The solution of Equation (A.6) is $N \geq 3$. That is, the absolute positions in three-dimensional coordinates of all transponders can be determined with as few as three consecutive range measurements from four fixed transponders.

Mikhail states the two-dimensional distance condition and its linearization (Mikhail, 1981, pp266-268), which can be extended to three-dimensional coordinates. - Equation (A.5) can be linearized:

$$\hat{d}_i = d_i^0 + [(X_i - U) / d_i^0] \Delta X_i + [(Y_i - V) / d_i^0] \Delta Y_i + [(Z_i - W) / d_i^0] \Delta Z_i \quad . \quad (A.7)$$

The adjusted distance is then:

$$\hat{d}_i = d_i + v_i \quad (A.8)$$

where d_i are observed values of the distance; and
 v_i are corresponding residuals.

Equation (A.7) then becomes:

$$v_i - [(X_i - U) / d_i^0] \Delta X_i - [(Y_i - V) / d_i^0] \Delta Y_i - [(Z_i - W) / d_i^0] \Delta Z_i = d_i^0 - d_i \quad (A.9)$$

Letting

$$f_i = d_i^0 - d_i \quad (A.10)$$

in matrix form Equation (A.9) becomes:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ \vdots \\ \vdots \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} & 0 & 0 & \cdots \\ 0 & 0 & 0 & b_{21} & b_{22} & b_{23} & 0 & \cdots \\ \vdots & \ddots \\ \vdots & \vdots \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} \Delta X_1 \\ \Delta Y_1 \\ \Delta Z_1 \\ \vdots \\ \Delta Z_4 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ \vdots \end{bmatrix} \quad (\text{A. 11})$$

which may be concisely written as:

$$v_{4N \times 1} + B_{4N \times 12} \Delta x_{12 \times 1} = f_{4N \times 1} \quad (\text{A. 12})$$

where

v is residual of the observation;
 B is numerical coefficient of the unknown variable correction;
 Δx is unknown variable correction; and
 f is numerical constant term.

The weight matrix can be derived from the standard deviation of the slant range, which discussed in the previous chapter. The least squares solution of Equation (A. 12) is (Mikhail, 1981, p259):

$$N = B^T W B \quad (\text{A. 13})$$

$$t = B^T W f \quad (\text{A. 14})$$

$$\Delta x = N^{-1} t \quad (\text{A. 15})$$

$$\hat{x} = x_o + \Delta x \quad (\text{A. 16})$$

$$v = f - B \Delta x \quad (\text{A. 17})$$

$$\sigma_x = [\hat{\sigma}_o^2 (B N^{-1} B^t)]^{1/2} \quad (A. 18)$$

$$\hat{\sigma}_o^2 = v^t W v / r \quad (A. 19)$$

$$r = N - 3 \quad (A. 20)$$

where

N is coefficient matrix of the normal equations,
 W is weight matrix,
 t is vector of 'constants' in the normal equations,
 Δx is vector of parameter corrections,
 x_o is vector of approximate value,
 \hat{x} is vector of adjusted value,
 σ_x is standard deviation of adjusted distance,
 $\hat{\sigma}_o^2$ is estimate of the reference variance,
 r is number of statical degrees of freedom,
 N is number of given transducer position, and
 3 is number of observations necessary to specify
 uniquely the model that underlies the adjustment
 problem.

Since during linearization we neglect all second and higher order terms, we must ensure that X is not significantly in error because of this approximation. Consequently, we must iterate the solution by using X as a new approximation and compute another correction. This procedure is repeated until the correction is insignificant. The final result is:

$$x_a \doteq \hat{x}_n + \Delta x_n \doteq \hat{x}_n \quad (A. 21)$$

where

n is total iteration,
 \hat{x}_n is adjusted value at n iteration,
 Δx_n is correction at n iteration can be neglected compare
 with previous correction or within tolerance
 correction, and
 x_a is true value of each variable.

Since errors in transducer's position exist, it now must be established how this error affects the accuracy of transponder positions. This is discussed below:

Let the standard deviations of transducer coordinates be $\pm \sigma_u$, $\pm \sigma_v$, $\pm \sigma_w$. Recall Equation (A.5) :

$$l_i = [(U - X_i)^2 + (V - Y_i)^2 + (W - Z_i)^2]^{1/2} \quad (\text{A.22})$$

The unknown coordinates X, Y, Z , are:

$$X_i = U - [l_i^2 - (V - Y_i)^2 - (W - Z_i)^2]^{1/2} \quad (\text{A.23})$$

$$Y_i = V - [l_i^2 - (W - Z_i)^2 - (U - X_i)^2]^{1/2} \quad (\text{A.24})$$

$$Z_i = W - [l_i^2 - (U - X_i)^2 - (V - Y_i)^2]^{1/2} \quad (\text{A.25})$$

Now assume

$$F1 = [l_i^2 - (V - Y_i)^2 - (W - Z_i)^2]^{1/2} \quad (\text{A.26})$$

$$F2 = [l_i^2 - (W - Z_i)^2 - (U - X_i)^2]^{1/2} \quad (\text{A.27})$$

$$F3 = [l_i^2 - (U - X_i)^2 - (V - Y_i)^2]^{1/2} \quad (\text{A.28})$$

So, the variances of Equations (A.23), (A.24) and (A.25) are (Mikhail, 1981, p.181):

$$\sigma_x^2 = \sigma_u^2 - (l_i/F1)^2 \sigma_y^2 + (V - Y_i/F1)^2 \sigma_z^2 + (W - Z_i/F1)^2 \sigma_w^2 \quad (\text{A.29})$$

$$\sigma_y^2 = \sigma_v^2 - (\bar{Y}_i/F2)^2 \sigma_x^2 + (\bar{W}-\bar{Z}_i/F2)^2 \sigma_w^2 + (\bar{U}-\bar{X}_i/F2)^2 \sigma_v^2 \quad (A.30)$$

$$\sigma_z^2 = \sigma_w^2 - (\bar{Y}_i/F3)^2 \sigma_x^2 + (\bar{U}-\bar{X}_i/F3)^2 \sigma_w^2 + (\bar{V}-\bar{Y}_i/F3)^2 \sigma_u^2 \quad (A.31)$$

and the position accuracy is:

$$\sigma_T = \pm (\sigma_x^2 + \sigma_y^2 + \sigma_z^2)^{1/2} \quad (A.32)$$

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